

Coordinated Motion Control of Multiple Autonomous Underwater Vehicles

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Abstract

Spawned by recent advances in technology, widespread attention has been focused on the coordination of multiple autonomous vehicles. In numerous mission scenarios, the concept of a group of agents cooperating to achieve a determined goal is very attractive when compared with the solution of one single, heavily equipped vehicle, as it exhibits better performance in terms of efficiency, flexibility and robustness, and can more effectively react and adapt itself to the environment in which it operates. Applications of coordinated control of multiple vehicles include microsatellite clusters, formation flying of unmanned aerial vehicles and automated highway systems.

In the field of ocean exploration there has been a surge of interest worldwide in the development of autonomous robots equipped with systems to steer them accurately and reliably in the harsh marine environment and allow them to collect data at the surface and underwater. The cooperation of multiple autonomous underwater vehicles (AUVs) yields several advantages and leads to safer, faster, and far more efficient ways of exploring the ocean frontier, especially in hazardous conditions.

The dynamics of underwater vehicles however are characterized by hydrodynamic effects that must necessarily be taken into account during the control design. Moreover, it is common for underwater vehicles to be underactuated, that is, to have fewer actuators than degrees-of-freedom. Motion control for this class of vehicles is especially challenging because most of these systems exhibit nonholonomic constraints.

As there are strong practical limitations to the flow of information among vehicles, which is severely restricted by the nature of the supporting communications network, one of the aims of formation control must be to reduce the frequency at which information is exchanged among the systems involved. This is especially true in the case of AUVs, since underwater communications and positioning rely heavily on acoustic systems, which are plagued with intermittent failures, latency, and multipath effects.

It is in this framework that this thesis proposes a decentralized control structure, based on Lyapunov techniques and graph theory, that explicitly takes into account both the complex nonlinear dynamics of the cooperating vehicles and the constraints imposed by the topology of the inter-vehicle communications network. For a single vehicle, the solution to the motion control problem is based on an inner loop controller, that regulates the actuators so that a given speed reference is followed, and an outer loop kinematic controller that adjusts the speed reference to make the vehicle track a "virtual target" moving along the desired path. Coordination between multiple vehicles is then achieved by parametrizing the path of each vehicle and regulating the speed of the virtual target so to synchronize the parametrization states. The discontinuous nature of inter-vehicle communication is taken into account by introducing a logic-based communication system that minimizes the need for data exchange. Stability and convergence of the resulting system, and the conditions under which they hold, are assessed through a rigorous mathematical approach. The performance of the devised control strategies is finally evaluated with computer simulations.

Keywords: Autonomous underwater vehicles, Coordinated path-following, Coordination control, Graph theory, Nonlinear control, Underactuated systems.

Resumo

Devido aos recentes desenvolvimentos tecnológicos, a coordenação de múltiplos veículos autónomos tem suscitado um enorme interesse no seio da comunidade científica. Em diversos cenários de missão, o conceito de um grupo de agentes a cooperarem mutuamente para atingir um determinado objectivo é muito atractivo quando comparado com a solução de um único veículo fortemente equipado, pois exibe melhor desempenho em termos de eficiência, flexibilidade e robustez, para além de ser capaz de reagir e adaptar-se mais eficazmente ao ambiente onde opera. As aplicações do controlo de múltiplos veículos incluem constelações de microsatélites, voo em formação de veículos aéreos não-tripulados e sistemas de transporte automatizados (AHS - Automated Highway Systems).

No campo da exploração oceânica, tem havido um interesse crescente no desenvolvimento de robôs autónomos equipados com sistemas de navegação capazes de guiá-los com precisão e fiabilidade num ambiente inóspite como o oceano, e permitir-lhes recolher informação tanto à superfície como no meio subaquático. A cooperação de múltiplos veículos submarinos autónomos (VSAs) apresenta várias características vantajosas que conduzem a métodos mais seguros, mais rápidos e mais eficientes de explorar o oceano, especialmente em condições adversas.

A dinâmica dos veículos submarinos é caracterizada por efeitos hidrodinâmicos que têm necessariamente de ser considerados na concepção do sistema de controlo. Outra característica comum a estes veículos é o facto de serem frequentemente sub-actuados, *i.e.*, possuírem um menor número de actuadores do que graus de liberdade. O controlo de movimento para este tipo de veículos é especialmente difícil uma vez que geralmente estes sistemas têm propriedades não-holonómicas.

Devido à existência de fortes limitações práticas na transmissão de informação entre veículos, restringida pelas características da rede de comunicação subjacente, um dos objectivos do controlo de formação é a redução da frequência a que ocorre a troca de informação entre os sistemas envolvidos. Isso torna-se especialmente crucial no caso dos VSAs, uma vez que a comunicação e o posicionamento no ambiente subaquático baseiam-se principalmente em sistemas acústicos, que sofrem de falhas intermitentes, latência e efeitos de multipercurso.

É neste contexto que a presente tese propõe uma estrutura de controlo descentralizada, baseada em técnicas de Lyapunov e na teoria dos grafos, que considera explicitamente a complexa dinâmica não-linear dos veículos a cooperar e os constrangimentos impostos pela topologia da rede de comunicação. Para um único veículo, a solução do problema de controlo de movimento baseia-se num controlador de malha interna, que regula os actuadores de forma a que uma dada velocidade de referência seja mantida, e num controlador cinemático da malha externa que ajusta a velocidade de referência de modo a que o veículo siga um alvo virtual que percorre o trajecto desejado.

A coordenação entre múltiplos veículos é obtida através da parametrização do trajecto de cada veículo e da regulação da velocidade do alvo virtual de forma a sincronizar os estados de parametrização. A natureza descontínua da comunicação entre veículos é tida em conta introduzindo um sistema de comunicação baseado em lógica que minimiza a necessidade de troca de informação. A estabilidade e a convergência do sistema resultante, e as condições em que são válidas, são analisadas através de um estudo matemático rigoroso. O desempenho das estratégias concebidas é finalmente avaliado por meio de simulações computacionais.

Palavras Chave: Controlo cooperativo, Controlo não-linear, Seguimento coordenado de caminhos, Sistemas sub-actuados, Teoria dos grafos, Veículos submarinos autónomos.

"You like the sea, Captain?"

"Yes; I love it! The sea is everything. It covers seven-tenths of the terrestrial globe. Its breath is pure and healthy. It is an immense desert, where man is never lonely, for he feels life stirring on all sides. The sea is only the embodiment of a supernatural and wonderful existence. It is nothing but love and emotion; it is the 'Living Infinite', as one of your poets has said. In fact, Professor, Nature manifests herself in it by her three kingdoms, mineral, vegetable, and animal. The sea is the vast reservoir of Nature. The globe began with sea, so to speak; and who knows if it will not end with it? In it is supreme tranquility. The sea does not belong to despots. Upon its surface men can still exercise unjust laws, fight, tear one another to pieces, and be carried away with terrestrial horrors. But at thirty feet below its level, their reign ceases, their influence is quenched, and their power disappears. Ah! Sir, live—live in the bosom of the waters! There only is independence! There I recognize no masters! There I am free!"

Jules Verne, 20,000 Leagues Under The Sea

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Introduction

1.1 Historical perspective

Recent technological advances have spurred a broad interest in autonomous, adaptable vehicle formations. The development of powerful control techniques for single vehicles, the explosion in computation and communication capabilities, and the advent of miniaturization technologies have raised interest in vehicles which can interact autonomously with the environment and other vehicles to perform, in the presence of uncertainty and adversity, tasks beyond the ability of individual vehicles. The concept is based on the idea that a monolithic structure can be distributed in an inexpensive network of vehicles, resulting in a significant improvement in efficiency, performance, reconfigurability and robustness, and in the emergence of new capabilities. The types of applications envisioned are numerous.

Spacecraft formation flying is involved for example in autonomous rendezvous and docking missions. Recently, considerable interest has been focused on microsatellite clusters that have the advantage, over large and complex single-purpose satellites, to expand functionality, distribute risk, and reduce cost. An example is the United States Air Force Research Laboratory (ARFL) *TechSat 21* mission (Martin and Kilberg, 2001), which is investigating the ability of a cluster of microsatellites (Fig. 1.1), orbiting in close formation and jointly processing the interferometric data, to perform high-resolution imaging.



Figure 1.1: TechSat 21 mission (source: Martin and Kilberg, 2001)



Figure 1.2: TPF interferometer (source: NASA)

Through this technique, known as *Sparse Aperture Radar* (SAR), the cluster forms a large coherent array that provides a rich set of independently sampled data. To obtain the same results with a single satellite would require an antenna larger than can be deployed.

The goal of the *Terrestrial Planet Finder* mission (TPF) is to detect Earth-like planets that orbit nearby stars and to study the composition of their atmosphere. One of the TPF architectures that is at the moment under study by NASA is a separated infrared interferometer (Park, 2001), consisting of four spacecrafts equipped with telescopes that send their collected light to a fifth combiner spacecraft (Fig. 1.2). The interferometer would have a virtual baseline of several kilometers, enabling the detection of the very faint infrared emission from the planets.

Further examples of spacecraft formation flying can be found in (Beard *et al.*, 2001; Mesbahi and Hadaegh, 2001).

Advances in avionics, GPS-based navigation, and flight control techniques have brought unmanned aerial vehicle (UAV) technology to a point where it is routinely used in commercial and military applications, leading to renewed interest in **UAV formation flight** (Fax, 2002). Applications of this technology include air-to-air refueling, military maneuvers and drag reduction via close formation flight (see Giuletti *et al.*, 2000, and the references therein).

Considerable work has been done in the field of **coordinated control of land robots** (Desai *et al.*, 1998; Ögren *et al.*, 2002; Ghabcheloo *et al.*, 2007). A broad international effort is being made for the development of an *Automated Highway System* (AHS) that would improve safety by avoiding collisions, and increase vehicle throughput, thus reducing traffic (McMillin and Sanford, 1998; Horowitz and Varaiya, 2000).

1.1.1 Formation control of marine craft

In a great number of mission scenarios multiple marine vehicles must work in cooperation. Underway replenishment (UNREP), or replenishment at sea (RAS), is a method of transferring fuel, munitions, personnel and stores from one ship to another while both vessels are moving (see Fig. 1.3). This operation involves the coordination in a leader-follower scheme of the two vessels, that have to move in a alongside kind of formation (Kyrkjebø and Pettersen, 2003; Kyrkjebø *et al.*, 2004).

Following the recent advances in marine technology there has been a surge of interest worldwide in the development of autonomous surface crafts (ASCs) and underwater vehicles (AUVs) capable of exploring the oceans to collect data. In many cases two or more vehicles are required to maintain a determined spatial formation. Fig. 1.4 illustrates the coordinated operation of the Infante AUV and Delfim ASC, both designed and built at the Instituto Superior Técnico of Lisbon in the scope of the ASIMOV project (Pascoal and *et al.*, 2000). In this scenario the AUV serves as a mobile sensor suite to acquire scientific data while the



Figure 1.3: Underway replenishment (source: http://www.iwo-jima.navy.mil)



Figure 1.4: The ASIMOV project: coordinated path-following of an AUV and an ASC (source: Pascoal and *et al.*, 2000)

ASC plays the role of a fast communication relay between the AUV and a support ship. High data rate underwater communications can best be achieved if the emitter and the receiver are aligned along the same vertical line, in order to avoid multipath effects, so the ASC and the AUV must follow exactly the same horizontal path, shifted in the vertical.

In numerous other scenarios there are several disadvantages in using one single vehicle: among them lack of robustness to single point failure and inefficiency due to the fact that the vehicle might need to wander significantly to collect rich enough data. A cooperative network of vehicles has the potential to overcome these limitations and can adapt its behaviour/configuration, both i) in response to the measured environment, in order to improve performance and optimize the detection and measurement of fields and features of particular interest, or ii) in case of failure of one of the vehicles. Furthermore, in a cooperative mission scenario, each vehicle may only be required to carry a single sensor making each of the vehicles in the formation less complex then a single heavily equipped vehicle, thus increasing the reliability of the ensemble (Aguiar and Pascoal, 2007a). Some examples of missions that require cooperation between multiple vehicles are (Pascoal *et al.*, 2005):

Image acquisition. An underwater vehicle carries a strong light source and illuminates the scenery around a second underwater vehicle that must follow a pre-determined path and acquire images for scientific purposes.

Fast acoustic coverage of the seabed. Two vehicles are required to maneuver above the seabed, at identical or different depths, along parallel paths, and map the sea bottom using two copies of the same suite of acoustic sensors. By requesting the vehicles to traverse identical paths so as to make the acoustic beam coverage overlap on the seabed, large areas can be covered in a short time. A similar scenario can be envisioned where the vehicles use a set of vision sensors to inspect the same scenery from two different viewpoints, to try and acquire three-dimensional images of the seabed.

Quest for hydrothermal vents. Underwater hydrothermal vents produce methane that does not dissolve quickly in the water. A fleet of underwater vehicles, each equipped with a methane sensor, can detect the source of a vent by computing on-line and following the gradient of methane concentration (see Fig. 1.5).

A more detailed introduction to this subject can be found, along with other examples, in (Stilwell and Bishop, 2000; Encarnação and Pascoal, 2001; Fossen, 2002; Skjetne *et al.*, 2002).

1.2 Problem statement

From a theoretical viewpoint, the problems that must be solved to achieve coordination of multiple vehicles cover a vast number of fields that include navigation, guidance, and



Figure 1.5: AUV formation searching for thermal vents (source: http://www.grex-project.eu)

control. This thesis focuses on coordinated path-following, where multiple vehicles are required to follow pre-specified spatial paths while keeping a desired inter-vehicle formation pattern. This subject poses different and unique challenges in each of the areas of application reviewed in the previous section. However, as pointed out in (Fax and Murray, 2004) several common threads can be found. Two in particular are worth stressing:

- i) In most cases (the main exceptions being in the area of aircraft control) the vehicles are coupled through the task they are required to accomplish together, but are otherwise dynamically decoupled, that is, the motion of one vehicle does not directly affect the others.
- ii) Decisions must be made by each vehicle using only limited information about the other vehicles, as communications are restricted by the nature of the supporting network and may be subject to uncertainty and transmission delay.

The highly distributed nature of the vehicles' sensing and actuation modules one one side, and the strong practical limitations to the flow of information among vehicles on the other, require the adoption of a new control paradigm that departs considerably from classical centralized control strategies, in which a single controller possesses all the information needed to achieve the desired control objectives (Ghabcheloo *et al.*, 2006*a*). For these reasons, there has been over the past few years a flurry of activity in the area of multi-agent networks with application to engineering and science problems. Namely, in such topics as parallel computing (Tsitsiklis and Athans, 1984), synchronization of oscillators (Sepulchre *et al.*, 2003; Papachristodoulou and Jadbabaie, 2005), collective behavior and flocking (Jadbabaie *et al.*, 2003), consensus (Lin *et al.*, 2007), multi-vehicle formation control (Egerstedt and Hu, 2001), asynchronous protocols (Fang *et al.*, 2005), and graph theory and graph connectivity (Kim and Mesbahi, 2006).

In spite of significant progress in all these areas, much work remains to be done to develop strategies capable of yielding robust performance of a fleet of vehicles in the presence of complex vehicle dynamics, severe communication constraints, and partial vehicle failures. These difficulties are specially challenging in the field of marine robotics for two main reasons:

- *i)* The dynamics of marine vehicles are often complex and cannot be simply ignored or drastically simplified for control design purposes.
- ii) Underwater communications and positioning rely heavily on acoustic systems, which are plagued with intermittent failures, latency, and multipath effects. These effects set tight limits on the effective communication bandwidths that can be achieved and introduce latency in the measurements that are exchanged among the vehicles.

It is in this framework that this thesis proposes a decentralized control structure where the dynamics of the cooperating vehicles and the constraints imposed by the topology and the nature of the inter-vehicle communications network are explicitly taken into account.

1.3 Previous work and contributions

1.3.1 Motion control of underactuated vehicles

For fully actuated systems, the problems of trajectory-tracking and path-following are now reasonably well understood. However, for underactuated autonomous vehicles, *i.e.*, systems with a smaller number of control inputs than the number of independent generalized coordinates, they are still active research topics. The study of these systems is motivated by the fact that it is usually costly and often impractical (due to weight, reliability, complexity, and efficiency considerations) to fully actuate autonomous vehicles (Aguiar and Hespanha, 2003). Typical examples of underactuated systems include robot manipulators, wheeled robots, walking robots, spacecraft, aircraft, helicopters, missiles, surface vessels, and underwater vehicles. The motion control problem for underactuated vehicles is especially challenging because most of these systems are not fully feedback linearizable and exhibit nonholonomic constraints. A class of underactuated vehicles that poses considerable challenging in control system design is the class of marine underactuated vehicles. These vehicles exhibit complex hydrodynamic effects that must necessarily be taken into account.

Encarnação *et al.* (2000) and Encarnação and Pascoal (2000) propose Lyapunov based control laws to solve the path-following problem for a single autonomous underactuated vehicle. The advantage of nonlinear control when compared with classical contol strategies is that it explicitly exploits the physical structure of the vehicles, instead of opposing it. The path-following problem is divided into a dynamic and a kinematic task, the latter consisting in making the Serret-Frenet frame $\{\mathcal{F}\}$ associated to the vehicle track a frame attached to the closest point on the path. This strategy exhibits severe limitations, as the initial position of the vehicle has to lie inside a tube around the path, the radius of which has to be smaller than the smallest radius of curvature present in that path.

The solution proposed in (Lapierre *et al.*, 2003*b*) lifts these restrictions by controlling explicitly the rate of progression of a "virtual target" to be tracked along the path, that is, the Serret-Frenet frame is not attached to the point on the path that is closest to the vehicle. Instead, the origin of $\{\mathcal{F}\}$ is made to evolve according to a conveniently defined control law, effectively yielding an extra control variable. The same strategy is adopted and refined in (Aguiar and Hespanha, 2003, 2004, 2007; Aguiar and Pascoal, 2007*b*), where parametric modeling uncertainties are considered.

Borrowing from these results, in this thesis we decouple the motion control problem, designing independently an inner-loop dynamic controller and an outer-loop kinematic controller that produces the speed reference for the inner loop. The reason behind this choice is that most autonomous underwater vehicles are equipped with an inner-loop controller that regulates the thrusters so that the surge speed and yaw rate follow a given reference. Although better results could be achieved, in terms of saturation and smoothness of the control signal, designing one single controller, decoupling the problem results in greater portability, as the kinematic control laws obtained can be applied to a wide range of AUVs. A second contribution is to consider the case in which only some elements of the kinematic and dynamic states are available to the controllers. There are two main practical motivations:

- *i)* The sway velocity sensors are very expensive, and it is therefore interesting to see how the performance of the control system is limited by their absence.
- *ii)* The approach adopted in deriving the control laws for the inner and outer loop imply that the inner-loop controller has no access to the time derivative of the reference speed.

We therefore design the trajectory-tracking and path-following controllers assuming, first, that there are no restrictions in terms of accessible variables, then introducing the limitations above and proving that the stability and convergence properties still hold under some reasonable assumptions.

1.3.2 Coordinated control of multiple AUVs

The results of (Encarnação *et al.*, 2000; Encarnação and Pascoal, 2000) are applied, in (Encarnação and Pascoal, 2001), to the coordination control of an autonomous surface craft (ASC) and an autonomous underwater vehicle (AUV). However, the strategy adopted is not easily generalized to more than two vehicles and requires the exchange of a large amount of information between them.

A more general approach to coordinated path-following can be found in (Egerstedt and Hu, 2001), where the formation is defined as the global minimum of a "rigid body constraint function", and in (Skjetne *et al.*, 2002, 2003; Lapierre *et al.*, 2003*a*). The common thread is to divide the problem in a motion control task, to be solved individually for every vehicle, each having access to a set of local measurements, and a dynamic assignment task, consisting in synchronizing the parametrization states that capture the along path distances between the vehicles. This strategy, that is also the one adopted in this thesis, results in decoupling path-following (in space) and inter-vehicle coordination (in time). Notice however that the aforementioned works do not consider the communication constraints imposed by the topology of the inter-vehicle communications network.

The topics of information flow and cooperation control of vehicle formations are addressed in (Fax and Murray, 2002a,b), that propose a methodology based on a framework that involves the concept of Graph Laplacian, a matrix representation of the graph associated with a given communication network. In particular, the results in (Fax and Murray, 2002a) show clearly how the Laplacian plays a key role in assessing stability of the behavior of the vehicles in a formation.

In (Ghabcheloo *et al.*, 2006*a*; Ghabcheloo, 2007) this methodology is used to obtain coordination laws that hold in case of communication losses and time-delays. Is it assumed, however, that the flow of information is continuous, even though it may exhibit intermittent interruptions. Borrowing from (Yook *et al.*, 2002; Xu and Hespanha, 2006), the work in (Aguiar and Pascoal, 2007*a*) addresses explicitly the fact that inter-vehicle communications do not occur in a continuous manner, but take place at discrete instants of time.

The third contribution of this thesis is then to propose an approach that aims at reducing the frequency at which information is exchanged among the systems involved, extending the results of (Aguiar and Pascoal, 2007*a*) and focusing on the simulation of formations with a higher number of vehicles. A subset of the results reported here were presented in (Vanni *et al.*, 2007*a*,*b*; Aguiar *et al.*, 2007*c*).

1.4 Thesis outline

The following is a brief description of the structure of this thesis.

- **Chapter 2** contains the theoretical preliminaries of nonlinear control and of graph theory that will be recalled along the thesis.
- **Chapter 3** describes the dynamics of the general model of AUV and of the Sirene. The dynamic and kinematic equations are given, first for six degrees of freedom and then for motion on the horizontal plane.
- **Chapter 4** derives dynamic and kinematic control laws for the motion control of the AUV and analyses the properties of the closed-loop system.
- Chapter 5 addresses the problem of coordination between multiple vehicles, both with continuous and discrete communication, and proposes a logic based strategy to tackle the problem of time-delays.
- **Chapter 6** illustrates the simulations that were run to test the control strategies devised in the previous chapters.
- **Chapter 7** summarizes the results obtained and suggests the directions of further investigation.

Mathematical Preliminaries

This chapter introduces the most important concepts and terminology used throughout the thesis. After defining the notation in Section 2.1, we give a brief overview of the theory of Lyapunov stability (Section 2.2) and of graph theory (Section 2.3).

2.1 Notation

The following notation will be used in the thesis. All mathematical variables are represented in italics. Lower case refers to scalars or elements of sets. When x is a scalar, |x| denotes its absolute value. Vectors are represented in lower case bold. **1** is the vector with all elements equal to one. x_i refers to the *i*th element of vector x, ||x|| to its norm. The norms used in this thesis are the class of *p*-norms defined by

$$\|\boldsymbol{x}\|_p = (|x_1|^p + \ldots + |x_n|^p)^{1/p}$$

and

$$\|\boldsymbol{x}\|_{\infty} = \max_{i} |x_{i}|,$$

the (essential) supremum norm

$$\|\boldsymbol{x}_{[t_0,\infty)}\| = \sup_{t \ge t_0} \|\boldsymbol{x}(t)\|$$

and the asymptotic norm

$$\|\boldsymbol{x}\|_a = \lim_{t \to \infty} \|\boldsymbol{x}(t)\|.$$

For simplicity of notation, except when explicitly stated, $\|\cdot\|$ denotes the euclidean 2-norm. However, due to equivalence of norms, and since the choice of a norm on \mathbb{R}^n does not affect the properties of a function, many of the results in this thesis are independent of the particular norm that is used.

Matrices are represented in upper case, with A_{ij} referring to the element occupying the *i*th row and *j*th column of A. I denotes the identity matrix, the dimension of which will be clear from the context. Given vector norms on K^m and K^n , the corresponding *induced*

norm on the space of m-by-n matrices is defined as

$$||A|| = \max\{||Ax|| : x \in K^n \land ||x|| = 1\}$$

In the special case of euclidean 2-norm and square matrices, the induced matrix norm is the *spectral norm*, or the largest singular value of A:

$$||A||_2 = \sqrt{\lambda_{max}(A^*A)}$$

where A^* denotes the conjugate transpose of A.

Calligraphed letters refer to sets or graphs, and $|\mathcal{G}|$ denotes the cardinality of the set \mathcal{G} .

2.2 Nonlinear systems theory

2.2.1 Introduction

In this section we recall some necessary ideas about stability of the equilibrium points of autonomous and nonautonomous nonlinear systems. The theorems and definitions reported are borrowed, with the exception of Definition 2.9, from (Khalil, 2002). The proofs are not reported.

2.2.2 Lipschitz functions

To ensure some properties of the initial-value problem

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(t, \boldsymbol{x}), \ \boldsymbol{x}(t_0) = \boldsymbol{x}_0$$

a key constraint that is imposed on the function f(t, x) is the Lipschitz condition

$$\|\boldsymbol{f}(t,\boldsymbol{x}) - \boldsymbol{f}(t,\boldsymbol{y})\| \le L \|\boldsymbol{x} - \boldsymbol{y}\|$$
(2.1)

A function satisfying inequality (2.1) for all (t, \boldsymbol{x}) and (t, \boldsymbol{y}) in some neighborhood of (t_0, \boldsymbol{x}_0) is said to be *Lipschitz* in \boldsymbol{x} , and the positive constant *L* is called a *Lipschitz constant*.

Definition 2.1. A function f(t, x) is

- locally Lipschitz in x on [a, b] × D ⊂ ℝ × ℝⁿ if each point x ∈ D has a neighborhood D₀ such that f satisfies (2.1) on [a, b] × D₀ with some Lipschitz constant L₀.
- Lipschitz in x on [a, b] × D ⊂ ℝ × ℝⁿ if each point x ∈ D has a neighborhood D₀ such that f satisfies (2.1) on [a, b] × D₀ with the same Lipschitz constant L.
- globally Lipschitz in x if it is Lipschitz in x on $[a, b] \times \mathbb{R}^n \subset \mathbb{R} \times \mathbb{R}^n$.

The Lipschitz property of a function is stronger than continuity and, as stated in the following lemmas, weaker than continuous differentiability.

Lemma 2.1. If f(t, x) and $[\partial f / \partial x](t, x)$ are continuous on $[a, b] \times D$, for some domain $D \subset \mathbb{R}^n$, then f is locally Lipschitz in x on $[a, b] \times D$.

Lemma 2.2. If $\mathbf{f}(t, \mathbf{x})$ and $[\partial \mathbf{f} / \partial \mathbf{x}](t, \mathbf{x})$ are continuous on $[a, b] \times \mathbb{R}^n$, then \mathbf{f} is globally Lipschitz in \mathbf{x} on $[a, b] \times \mathbb{R}^n$ if and only if $[\partial \mathbf{f} / \partial \mathbf{x}]$ is uniformly bounded on $[a, b] \times \mathbb{R}^n$.

2.2.3 Lyapunov stability

Suppose the autonomous¹ system

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) \tag{2.2}$$

has an equilibrium point, which is assumed to be at the origin of \mathbb{R}^n , that is, $f(\mathbf{0}) = \mathbf{0}$. There is no loss of generality in doing so because any equilibrium point can be shifted to the origin via a change of variables.

Definition 2.2. The equilibrium point x = 0 of (2.2) is

• stable if, for each $\epsilon > 0$, there is $\delta = \delta(\epsilon) > 0$ such that

$$\|\boldsymbol{x}(0)\| < \delta \Rightarrow \|\boldsymbol{x}(t)\| < \epsilon, \ \forall \ t \ge 0$$

- unstable if it is not stable,
- asymptotically stable if it is stable and δ can be chosen such that

$$\|\boldsymbol{x}(0)\| < \delta \Rightarrow \lim_{t \to \infty} \boldsymbol{x}(t) = 0$$

In order to demonstrate that the origin is a stable equilibrium point, for each selected value of ϵ one must produce a value of δ , possibly dependent on ϵ , such that a trajectory starting in a δ neighborhood of the origin will never leave the ϵ neighborhood. It is possible to determine stability by examining the derivatives of some particular functions, without having to know explicitly the solution of (2.2).

Theorem 2.3 (Lyapunov's stability theorem). Let $\mathbf{x} = \mathbf{0}$ be an equilibrium point for (2.2) and $D \subset \mathbb{R}^n$ be a domain containing $\mathbf{x} = \mathbf{0}$. Let $V : D \to \mathbb{R}$ be a continuously differentiable function such that

$$V(\mathbf{0}) = 0 \text{ and } V(\mathbf{x}) > 0 \text{ in } D - \{0\}$$
(2.3)

$$V(\boldsymbol{x}) \le 0 \text{ in } D \tag{2.4}$$

¹A system in which function f does not depend explicitly on time.

Then, $\mathbf{x} = \mathbf{0}$ is stable. Moreover, if

$$\dot{V}(\boldsymbol{x}) < 0 \text{ in } D - \{0\}$$

then $\mathbf{x} = \mathbf{0}$ is asymptotically stable.

A function $V(\mathbf{x})$ satisfying condition (2.3) is said to be *positive definite*. If it satisfies the weaker condition $V(\mathbf{x}) \ge 0$ for $\mathbf{x} \ne \mathbf{0}$ it is said to be *positive semidefinite*. A function $V(\mathbf{x})$ is said to be *negative definite* or *negative semidefinite* if $-V(\mathbf{x})$ is *positive definite* or *positive semidefinite*, respectively. A continuously differentiable function $V(\mathbf{x})$ satisfying (2.3) and (2.4) is called a *Lyapunov function*, after the russian mathematician who laid the bases of this theory.

Theorem 2.4. Let x = 0 be an equilibrium point for (2.2). Let $V : \mathbb{R}^n \to \mathbb{R}$ be a continuously differentiable function such that

$$V(\mathbf{0}) = 0 \text{ and } V(\mathbf{x}) > 0, \ \forall \ \mathbf{x} \neq \mathbf{0}$$

$$(2.5)$$

$$\|\boldsymbol{x}\| \to \infty \Rightarrow V(\boldsymbol{x}) \to \infty \tag{2.6}$$

$$V(\boldsymbol{x}) < 0, \quad \forall \ \boldsymbol{x} \neq \boldsymbol{0} \tag{2.7}$$

then x = 0 is globally asymptotically stable.

A function satisfying condition (2.6) is said to be *radially unbounded*.

2.2.4 The invariance principle

The stability theorems of Section 2.2.3 require to find a Lyapunov function whose time derivative is negative definite. If in a domain about the origin, however, a Lyapunov function can be found whose derivative along the trajectories of the system is only negative semidefinite, asymptotic stability of the origin might still be proved, provided that no trajectory can stay identically at the points where $\dot{V}(\boldsymbol{x}) = 0$, except at the origin. This idea follows from LaSalle's *invariance principle*, which is not enunciated here, since a few definitions would require to be introduced to state the theorem. Instead, we report two corollaries which had been proved before the introduction of the invariance principle and whose results are more relevant in the scope of this thesis.

Theorem 2.5 (Barbashin's theorem). Let $\mathbf{x} = \mathbf{0}$ be an equilibrium point for (2.2). Let $V: D \to \mathbb{R}$ be a continuously differentiable positive definite function on a domain $D \subset \mathbb{R}^n$ containing the origin $\mathbf{x} = \mathbf{0}$, such that $\dot{V}(\mathbf{x}) \leq 0$ in D. Let $S = \{\mathbf{x} \in D | \dot{V}(\mathbf{x}) = 0\}$ and suppose that no solution can stay identically in S, other than the trivial solution $\mathbf{x}(t) \equiv 0$. Then, the origin is asymptotically stable.

Theorem 2.6 (Krasovskii's theorem). Let $\mathbf{x} = \mathbf{0}$ be an equilibrium point for (2.2). Let $V : \mathbb{R}^n \to \mathbb{R}$ be a continuously differentiable, radially unbounded, positive definite function such that $\dot{V}(\mathbf{x}) \leq 0$ for all $\mathbf{x} \in \mathbb{R}^n$. Let $S = \{\mathbf{x} \in \mathbb{R}^n | \dot{V}(\mathbf{x}) = 0\}$ and suppose that no solution can stay identically in S, other than the trivial solution $\mathbf{x}(t) \equiv 0$. Then, the origin is globally asymptotically stable.

2.2.5 Nonautonomous systems

The notions of stability and asymptotic stability of equilibrium points of nonautonomous systems are very similar to those introduced in Definition (2.2) for autonomous systems. The difference is that while the solution of an autonomous system depends only on $(t - t_0)$, the solution of the nonautonomous system

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(t, \boldsymbol{x}), \ \boldsymbol{x}(t_0) = \boldsymbol{x}_0 \tag{2.8}$$

depends on both t and t_0 , so stability and asymptotic stability need to be redefined as uniform properties with respect to the initial time. The origin is an equilibrium point of (2.8) at t = 0 if

$$\boldsymbol{f}(t, \boldsymbol{0}) = \boldsymbol{0}, \ \forall \ t \ge 0$$

Again, there is no loss of generality since an equilibrium point at the origin could be a translation of a nonzero equilibrium point.

Definition 2.3. The equilibrium point x = 0 of (2.8) is

• stable if, for each $\epsilon > 0$, there is $\delta = \delta(\epsilon, t_0) > 0$ such that

$$\|\boldsymbol{x}(t_0)\| < \delta \Rightarrow \|\boldsymbol{x}(t)\| < \epsilon, \ \forall \ t \ge t_0 \ge 0$$
(2.9)

- uniformly stable if, for each $\epsilon > 0$, there is $\delta = \delta(\epsilon) > 0$, independent of t_0 , such that (2.9) is satisfied.
- unstable if it is not stable.
- asymptotically stable if it is stable and there is a positive constant $c = c(t_0)$ such that $\mathbf{x}(t) \to \mathbf{0}$ as $t \to \infty, \forall ||\mathbf{x}(t_0)|| < c$.
- uniformly asymptotically stable if it is uniformly stable and there is a positive constant c, independent of t₀, such that for all ||**x**(t₀)|| < c, **x**(t) → **0** as t → ∞ uniformly in t₀; that is, for each η > 0, there is T = T(η) > 0 such that

$$\|\boldsymbol{x}(t)\| < \eta, \ \forall \ t \ge t_0 + T(\eta), \ \forall \ \|\boldsymbol{x}(t_0)\| < c$$

 globally uniformly asymptotically stable if it is uniformly stable, δ(ε) can be chosen to satisfy lim_{ε→∞} δ(ε) = ∞, and, for each pair of positive numbers η and c, there is T = T(η, c) > 0 such that

$$\|\boldsymbol{x}(t)\| < \eta, \ \forall \ t \ge t_0 + T(\eta, c), \ \forall \ \|\boldsymbol{x}(t_0)\| < c$$

Equivalent definitions, more convenient to the approach followed in this thesis to deal with the different control problems, can be given using two comparison functions, known as class \mathcal{K} and \mathcal{KL} functions.

Definition 2.4. A continuous function $\alpha : [0, a) \to [0, \infty)$ is said to belong to class \mathcal{K} if it is strictly increasing and $\alpha(0) = 0$. It is said to belong to class \mathcal{K}_{∞} if $a = \infty$ and $\alpha(r) \to \infty$ as $r \to \infty$.

Definition 2.5. A continuous function $\beta : [0, a) \times [0, \infty) \to [0, \infty)$ is said to belong to class \mathcal{KL} if, for each fixed s, the mapping $\beta(r, s)$ belongs to class \mathcal{K} with respect to r and, for each fixed r, the mapping $\beta(r, s)$ is decreasing with respect to s and $\beta(r, s) \to 0$ as $s \to \infty$.

The next lemma redefines uniform stability and uniform asymptotic stability using class \mathcal{KL} functions.

Lemma 2.7. The equilibrium point x = 0 of (2.8) is

 uniformly stable if and only if there exist a class K function α and a positive constant c, independent of t₀, such that

$$\|\boldsymbol{x}(t)\| < \alpha(\|\boldsymbol{x}(t_0)\|), \ \forall \ t \ge t_0 \ge 0, \ \forall \ \|\boldsymbol{x}(t_0)\| < c$$

• uniformly asymptotically stable if and only if there exist a class \mathcal{KL} function β and a positive constant c, independent of t_0 , such that

$$\|\boldsymbol{x}(t)\| < \beta(\|\boldsymbol{x}(t_0)\|, t - t_0), \ \forall \ t \ge t_0 \ge 0, \ \forall \ \|\boldsymbol{x}(t_0)\| < c$$
(2.10)

• globally uniformly asymptotically stable if and only if inequality (2.10) is satisfied for any initial state $\boldsymbol{x}(t_0)$.

A special case of uniform asymptotic stability arises when the class \mathcal{KL} function β in (2.10) takes the form $\beta(r, s) = kre^{-\lambda s}$.

Definition 2.6. The equilibrium point $\mathbf{x} = \mathbf{0}$ of (2.8) is exponentially stable if there exist positive constants c, k and λ such that

$$\|\boldsymbol{x}(t)\| \le k \|\boldsymbol{x}(t_0)\| e^{-\lambda(t-t_0)}, \ \forall \ \|\boldsymbol{x}(t_0)\| < c$$
(2.11)

and globally exponentially stable if (2.11) is satisfied for any initial state $\mathbf{x}(t_0)$.

Lyapunov theory for autonomous systems can be extended to nonautonomous systems. The following theorems concentrate on uniform stability, uniform asymptotic stability and exponential stability.

Theorem 2.8. Let $\mathbf{x} = \mathbf{0}$ be an equilibrium point for (2.8) and $D \subset \mathbb{R}^n$ be a domain containing $\mathbf{x} = \mathbf{0}$. Let $V : [0, \infty) \times D \to \mathbb{R}$ be a continuously differentiable function such that

$$W_1(\boldsymbol{x}) \le V(t, \boldsymbol{x}) \le W_2(\boldsymbol{x}) \tag{2.12}$$

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial \boldsymbol{x}} \boldsymbol{f}(t, \boldsymbol{x}) \le 0$$
(2.13)

 $\forall t \geq 0 \text{ and } \forall x \in D, \text{ where } W_1(x) \text{ and } W_2(x) \text{ are continuous positive definite functions}$ on D. Then, x = 0 is uniformly stable.

Theorem 2.9. Suppose the assumptions of Theorem (2.8) are satisfied with inequality (2.13) strengthened to

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial \boldsymbol{x}} \boldsymbol{f}(t, \boldsymbol{x}) \leq -W_3(\boldsymbol{x})$$

 $\forall t \geq 0 \text{ and } \forall x \in D, \text{ where } W_3(x) \text{ is a continuous positive definite function on } D.$ Then, $x = \mathbf{0}$ is uniformly asymptotically stable. Moreover, if positive constants r and c are chosen such that $B_r = \{ \|x\| \leq r \} \subset D$ and $c < \min_{\|x\|=r} W_1(x)$, then every trajectory starting in $\{x \in B_r \mid W_2(x) \leq c\}$ satisfies

$$\|\boldsymbol{x}(t)\| \leq \beta(\|\boldsymbol{x}(t_0)\|, t-t_0), \ \forall \ t \geq t_0 \geq 0$$

for some class \mathcal{KL} function β . Finally, if $D = \mathbb{R}^n$ and $W_1(\mathbf{x})$ is radially unbounded, then $\mathbf{x} = \mathbf{0}$ is globally uniformly asymptotically stable.

Theorem 2.10. Let $\mathbf{x} = \mathbf{0}$ be an equilibrium point for (2.8) and $D \subset \mathbb{R}^n$ be a domain containing $\mathbf{x} = \mathbf{0}$. Let $V : [0, \infty) \times D \to \mathbb{R}$ be a continuously differentiable function such that

$$egin{aligned} &k_1 \|m{x}\|^a \leq V(t,m{x}) \leq k_2 \|m{x}\|^a \ &rac{\partial V}{\partial t} + rac{\partial V}{\partial m{x}}m{f}(t,m{x}) \leq -k_3 \|m{x}\|^a \end{aligned}$$

 $\forall t \geq 0 \text{ and } \forall x \in D$, where k_1 , k_2 , k_3 and a are positive constants. Then, $x = \mathbf{0}$ is exponentially stable. If the assumptions hold globally, then $x = \mathbf{0}$ is globally exponentially stable.

2.2.6 Boundedness

Even when there is no equilibrium point at the origin, Lyapunov analysis can be used to show boundedness of the solution of the nonlinear system

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(t, \boldsymbol{x}) \tag{2.14}$$

Definition 2.7. The solutions of (2.14) are

 uniformly bounded if there exists a positive constant c, independent of t₀ ≥ 0, and for every a ∈ (0, c), there is β = β(a) > 0, independent of t₀, such that

$$\|\boldsymbol{x}(t_0)\| \le a \Rightarrow \|\boldsymbol{x}(t)\| \le \beta, \ \forall \ t \ge t_0$$

$$(2.15)$$

- globally uniformly bounded if (2.15) holds for arbitrarily large a.
- uniformly ultimately bounded with ultimate bound b if there exist positive constants b and c, independent of $t_0 \ge 0$, and for every $a \in (0,c)$, there is $T = T(a,b) \ge 0$, independent of t_0 , such that

$$\|\boldsymbol{x}(t_0)\| \le a \Rightarrow \|\boldsymbol{x}(t)\| \le b, \ \forall \ t \ge t_0 + T$$

$$(2.16)$$

• globally uniformly ultimately bounded if (2.16) holds for arbitrarily large a.

The word "uniform" is omitted for autonomous systems since the solution depends only on $t - t_0$.

2.2.7 Input-to-state stability

Consider the system

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(t, \boldsymbol{x}, \boldsymbol{u}) \tag{2.17}$$

where $\boldsymbol{f} : [0, \infty) \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is piecewise continuous in t and locally Lipschitz in \boldsymbol{x} and \boldsymbol{u} and the input $\boldsymbol{u}(t)$ is piecewise continuous. Suppose the unforced system

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(t, \boldsymbol{x}, \boldsymbol{0}) \tag{2.18}$$

has a globally uniformly asymptotically stable equilibrium point at the origin $\boldsymbol{x} = \boldsymbol{0}$. It is possible to view system (2.17) as a perturbation of the unforced system (2.18). Under certain conditions if the input $\boldsymbol{u}(t)$ is bounded, that is, its supremum norm $\|\boldsymbol{u}_{[t_0,\infty)}\|$ is finite, the state $\boldsymbol{x}(t)$ is also bounded.



Figure 2.1: A graphical representation of the asymptotic gain property

Definition 2.8. The system (2.17) is said to be input-to-state stable if there exist a class \mathcal{KL} function β and a class \mathcal{K} function γ such that for any initial state $\mathbf{x}(t_0)$ and any bounded input $\mathbf{u}(t)$, the solution $\mathbf{x}(t)$ exists for all $t \geq t_0$ and satisfies

$$\|\boldsymbol{x}(t)\| \le \beta(\|\boldsymbol{x}(t_0)\|, t - t_0) + \gamma(\|\boldsymbol{u}_{[t_0, t]}\|)$$
(2.19)

The notion of input-to-state stability was first introduced in (Sontag, 1989). Inequality (2.19) guarantees that for any bounded input $\boldsymbol{u}(t)$ the state $\boldsymbol{x}(t)$ will be bounded. Furthermore, as t increases, the state $\boldsymbol{x}(t)$ will be ultimately bounded by a class \mathcal{K} function of $\|\boldsymbol{u}_{[t_0,\infty)}\|$, that is, an input-to-state stable system satisfies the *asymptotic gain property*: there is some class \mathcal{K}_{∞} function γ such that

$$\|\boldsymbol{x}(t)\|_{a} \leq \gamma \left(\|\boldsymbol{u}_{[t_{0},\infty)}\|\right)$$

In other words (see Fig. 2.1) for all large enough t, the trajectory exists, and it gets arbitrarily close to a sphere whose radius is proportional, in a possibly nonlinear way quantified by the function γ , to the amplitude of the input (Sontag, 2006). Observe that, since only large values of t matter in the limit, one can equally well consider merely tails of the input u when computing its supremum norm:

$$\|\boldsymbol{x}(t)\|_{a} \leq \gamma \left(\|\boldsymbol{u}(t)\|_{a}\right)$$

The imediate consequence is that if u(t) converges to zero as $t \to \infty$, so does x(t). The

following theorems give sufficient conditions for input-to-state stability.

Theorem 2.11. Let $V : [0, \infty) \times \mathbb{R}^n \to \mathbb{R}$ be a continuously differentiable function such that

$$\alpha_1(\|\boldsymbol{x}\|) \le V(t, \boldsymbol{x}) \le \alpha_2(\|\boldsymbol{x}\|) \tag{2.20}$$

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial \boldsymbol{x}} \boldsymbol{f}(t, \boldsymbol{x}, \boldsymbol{u}) \le -W_3(\boldsymbol{x}), \ \forall \ \|\boldsymbol{x}\| \ge \rho(\|\boldsymbol{u}\|) > 0$$
(2.21)

 $\forall (t, \boldsymbol{x}, \boldsymbol{u}) \in [0, \infty) \times \mathbb{R}^n \times \mathbb{R}^m$, where α_1 and α_2 are class \mathcal{K}_{∞} functions, ρ is a class \mathcal{K} function, and $W_3(\boldsymbol{x})$ is a continuous positive definite function on \mathbb{R}^n . Then, the system (2.17) is input-to-state stable with $\gamma = \alpha_1^{-1} \circ \alpha_2 \circ \rho$.

For autonomous systems conditions (2.20) and (2.21) are also necessary. It is common in literature to abbreviate input-to-state stability as ISS and to call function V of Theorem (2.11) an ISS-Lyapunov function.

Theorem 2.12. Suppose f(t, x, u) is continuously differentiable and globally Lipschitz in (x, u), uniformly in t. If the unforced system (2.18) has a globally exponentially stable equilibrium point in x = 0, then the system (2.17) is input-to-state stable.

An alternative definition of ISS was suggested in (Sontag and Wang, 1995), substituting inequality (2.19) with

$$\|\boldsymbol{x}(t)\| \le \max\{\beta(\|\boldsymbol{x}(t_0)\|, t - t_0), \gamma(\|\boldsymbol{u}_{[t_0, t]}\|)\}$$
(2.22)

Definition 2.22 and Definition 2.8 are equivalent, with the expressions of β and γ being, in general, different. In some cases inequality (2.22) cannot be satisfied for any $\boldsymbol{x}(t_0)$ and $\boldsymbol{u}(t)$. To address these class of problems, in (Teel, 1996) the concept of *ISS with restrictions* on the initial states and inputs was introduced.

Definition 2.9. The system (2.17) is said to be input-to-state stable with restrictions $X \subset \mathbb{R}^n$ and $\Delta > 0$ on the initial state $\mathbf{x}(t_0)$ and the input \mathbf{u} respectively if there exist a class \mathcal{KL} function β and a class \mathcal{K} function γ such that for any initial state $\mathbf{x}(t_0) \in X$ and any bounded input function $\mathbf{u}(t)$ satisfying $\|\mathbf{u}_{[t_0,\infty)}\| < \Delta$ the solution $\mathbf{x}(t)$ exists for all $t \geq t_0$ and satisfies

$$\|\boldsymbol{x}(t)\| \le \max\{\beta(\|\boldsymbol{x}(t_0)\|, t-t_0), \gamma(\|\boldsymbol{u}_{[t_0,\infty)}\|)\}$$

To distinguish between this last definition and Definition 2.8 some authors refer to the latter as to *global ISS*. In the remainder of this thesis however, and with no ambiguity, the two definitions will be referred to as ISS stability and ISS stability with restrictions.



Figure 2.2: An undirected graph \mathcal{G} and a directed graph \mathcal{G}'

2.3 Graph theory

2.3.1 Introduction

Coordinated motion control requires the vehicles to exchange information about their states over a communications network. In general no vehicle can communicate directly with every other vehicle in the formation. The topology of the communications network must therefore be addressed explicitly. Graph theory is a useful tool to model and study the impact of communication topologies on the performance of the coordinated system. This section contains a review of the basic concepts that will be used in the thesis. The definitions and theorems reported borrow from (Diestel, 2005) for undirected graphs, which are suited to model bi-directional communication networks, and from (Fax, 2002; Lin *et al.*, 2005) for directed graphs, used to model uni-directional communication networks.

2.3.2 Basic definitions

An undirected graph (or simply graph) is a pair $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ of sets that satisfy $\mathcal{E} \subset \mathcal{V}^2$, that is, the elements of $\mathcal{E}(\mathcal{G})$ are 2-element subsets of $\mathcal{V}(\mathcal{G})$. The elements of \mathcal{V} are the vertices (or nodes, or points) of the graph \mathcal{G} ; the elements of \mathcal{E} are its edges (or lines). The number of vertices of a graph \mathcal{G} is its order, written as $|\mathcal{G}|$; its number of edges is denoted by $||\mathcal{G}||$. A vertex $v \in \mathcal{V}$ is incident with an edge $e \in \mathcal{E}$ if $v \in e$; then e is an edge at v. The set of all the edges in \mathcal{E} at a vertex v is denoted $\mathcal{E}(v)$. The degree of a vertex v is the number $|\mathcal{E}(v)|$ of edges at v. A vertex of degree 0 is isolated. An edge $\{x, y\}$ is denoted by xy or yx; x and y are its ends. In a graph, the ends of an edge are always different, and no two edges have the same ends, that is, there are no loops or multiple edges. For this reason sometimes graphs are referred to as simple graphs to distinguish them from multigraphs, that can have loops and multiple edges. Two edges $x, y \in \mathcal{V}(\mathcal{G})$ are adjacent, or neighbors, if $xy \in \mathcal{E}(\mathcal{G})$. Two edges $e \neq f$ are adjacent if they have an end in common. If all the vertices of \mathcal{G} are pairwise adjacent, then \mathcal{G} is complete. A complete graph on n vertices is a \mathcal{K}^n . Pairwise non adjacent vertices or edges are called independent.



Figure 2.3: \mathcal{G}' is an induced subgraph of \mathcal{G} , \mathcal{G}'' is a spanning subgraph, but not an induced one, of \mathcal{G}



Figure 2.4: A path $\mathcal{P} = \mathcal{P}^3$ and a cycle $\mathcal{C} = \mathcal{C}^4$

If the graph is *directed* (or a *digraph*) (see Fig. 2.2) the elements of \mathcal{E} are called *arcs*, or directed edges. The first element of the arc a = xy is denoted tail(a), the second is denoted head(a). It is said that a points from x to y. If $tail(a) \neq head(a)$ and each element of \mathcal{E} is unique, the graph is called *oriented*. The *in(out)-degree* of a vertex v is the number of arcs with v as its head (tail). A directed graph is complete if every possible arc exists.

Define $\mathcal{G} \cup \mathcal{G}' = (\mathcal{V} \cup \mathcal{V}', \mathcal{E} \cup \mathcal{E}')$ and $\mathcal{G} \cap \mathcal{G}' = (\mathcal{V} \cap \mathcal{V}', \mathcal{E} \cap \mathcal{E}')$. If $\mathcal{G} \cap \mathcal{G}' = \emptyset$, \mathcal{G} and \mathcal{G}' are *disjoint*. If $\mathcal{V}' \subseteq \mathcal{V}$ and $\mathcal{E}' \subseteq \mathcal{E}$, then \mathcal{G}' is a *subgraph* of \mathcal{G} (and \mathcal{G} is a *supergraph* of \mathcal{G}'), written $\mathcal{G}' \subseteq \mathcal{G}$. If \mathcal{E}' contains contains every arc (edge) in \mathcal{E} whose head and tail (ends) are in \mathcal{V}' , then \mathcal{G}' is termed an *induced subgraph* of \mathcal{G} , and \mathcal{V}' is said to *induce* or *span* \mathcal{G}' in \mathcal{G} (see Fig. 2.3). If $\mathcal{G}' \subseteq \mathcal{G}$ and $\mathcal{V}' = \mathcal{V}$ then \mathcal{G}' is a *spanning subgraph* of \mathcal{G} .

2.3.3 Connectivity

A path from x_0 to x_n is a non-empty graph $\mathcal{P} = (\mathcal{V}, \mathcal{E})$ of the form

$$\mathcal{V} = \{x_0, x_1, \dots, x_k\}$$
 $\mathcal{E} = \{x_0 x_1, x_1 x_2, \dots, x_{k-1} x_k\}.$

The node x_0 has access to x_k , or equivalently x_k is said to be reachable from x_0 . The vertices x_1, \ldots, x_{k-1} are the *inner* vertices of \mathcal{P} . If a node is reachable from any other node then it is globally reachable. The number of arcs (edges) of a path is its *length*, the path of length k being denoted by \mathcal{P}^k . Two or more paths are *independent* if none of them contains



Figure 2.5: Graph \mathcal{G} is strongly connected, \mathcal{G}' is connected and \mathcal{G}'' is disconnected

an inner vertex of another. The *distance* in \mathcal{G} of two vertices x and y is the length of the shortest path from x to y in \mathcal{G} . If no such path exists, the distance is infinite. The greatest distance between any two vertices of \mathcal{G} is the *diameter* of \mathcal{G} . If $\mathcal{P}^k = (\mathcal{V}, \mathcal{E})$ is a path with $k \geq 3$ from x_0 to x_k , then the graph $\mathcal{C} = (\mathcal{V}, \mathcal{E}')$ with $\mathcal{E}' = E + x_k x_0$ is called a *cycle*. The *length* of a cycle is its number of edges (or vertices); the cycle of length k is called a *k-cycle* and denoted by \mathcal{C}^k (see Fig. 2.4).

Two vertices which have access to one another are said to *communicate*. A non-empty graph \mathcal{G} is called *strongly connected* if all vertices communicate with each other or, equivalently, if every vertex is globally reachable. A graph in which disjoint subsets of vertices exists whose elements do not have access to one another is termed *disconnected* (Fig. 2.5). An undirected graph is either strongly connected or disconnected. Communication is an equivalence relation, and the equivalence classes of \mathcal{V} induced by the communication relation are termed *components* of \mathcal{G} (Fig. 2.6). A component, being connected, is always non-empty, and the empty graph has no components. An *acyclic* graph, one not containing any cycles, is called a *forest*, a connected forest is called a *tree*.

Theorem 2.13. A connected graph with n vertices is a tree if and only if it has n-1 edges.

The vertices of degree 1 in a tree are its *leaves*. Every connected graph contains a spanning tree (any minimal connected spanning subgraph).

2.3.4 Algebraic graph theory

If an arbitrary enumeration is assigned to its vertices and edges (or arcs), matrices can be associated to a graph to represent it. Algebraic graph theory studies the relationships between the structure and the properties of a graph and its matricial representations. The *adjacency matrix* $A(\mathcal{G})$ of an undirected graph is the symmetric square matrix of size $|\mathcal{G}|$, defined by $A_{ij} = 1$ if $v_i v_j \in \mathcal{E}$, and $A_{ij} = 0$ otherwise. The definition holds for directed graphs, keeping in mind that $v_i v_j \neq v_j v_i$, that is, $A_{ij} = 1$ if v_i is the tail of an arc going to v_j , so in general A is not symmetric. The adjacency matrix uniquely defines a graph,



Figure 2.6: A graph with four components, and a spanning tree in each component

although for a given graph A is not itself unique, as it depends on the enumeration of the vertices. The *degree matrix* of an undirected (directed) graph is the square matrix $D(\mathcal{G})$ in which the elements of the diagonal are the degrees (out-degrees) of the corresponding vertices, that is, $D_{ii} = |\mathcal{A}(v_i)|$. For an oriented graph the *incidence matrix* can be defined as the matrix $M(\mathcal{G})$ with rows and columns indexed by the vertices and arcs of \mathcal{G} , with elements

$$m_{ij} = \begin{cases} +1, & \text{if } v_i \text{ is the head of arc } a_j \\ -1, & \text{if } v_i \text{ is the tail of arc } a_j \\ 0, & \text{otherwise} \end{cases}$$

The Laplacian of a graph is defined² as

$$L = (D - A) \tag{2.23}$$

By construction, L is positive semi-definite. If \mathcal{G} is undirected then L is also symmetric, so

Theorem 2.14. If \mathcal{G} is undirected, then all eigenvalues of L are real.

Observe that the rows of L sum to zero by definition. Then

Theorem 2.15. Zero is an eigenvalue of L, and 1 is the corresponding right eigenvector.

A further result is that

Theorem 2.16. If \mathcal{G} is strongly connected, the zero eigenvalue of L is simple. If it is not, the multiplicity m of the zero eigenvalue is equal to the number of final components of \mathcal{G} . The kernel of L has dimension m, and is spanned by a basis of m nonnegative vectors.

Consider now the normalized Laplacian

$$L_D = D^{-1}(D - A) (2.24)$$

²An equivalent definition valid for oriented graphs is $L = MM^{T}$.
with $D_{ii}^{-1} = 0$ by definition when $D_{ii} = 0$, *i.e.*, when a vertex is isolated; a result obtained applying the Perron-Frobenius theory is (Fax, 2002)

Theorem 2.17. All eigenvalues of L_D lie in a disk of radius 1 centered at the point 1 + 0j in the complex plane, denoted the Perron disk. If \mathcal{G} is strongly connected and aperiodic, all nonzero eigenvalues lie in the interior of the Perron disk. If \mathcal{G} is k-periodic, L_D has k evenly spaced eigenvalues on the boundary of the Perron disk.

Example. Consider the undirected graph \mathcal{G} and the directed graph \mathcal{G}' in Fig. 2.2. Then

$A(\mathcal{G}) = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$	$A(\mathcal{G}') = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0$
$D(\mathcal{G}) = \begin{bmatrix} 3 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$	$D(\mathcal{G}') = \begin{bmatrix} 3 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 &$
$L(\mathcal{G}) = \begin{bmatrix} 3 & -1 & -1 & -1 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 2 & -1 & 0 \\ -1 & 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$	$L(\mathcal{G}') = \begin{bmatrix} 3 & -1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0$
$L_D(\mathcal{G}) = \begin{bmatrix} 1 & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & 0\\ -1 & 1 & 0 & 0 & 0\\ -\frac{1}{2} & 0 & 1 & -\frac{1}{2} & 0\\ -\frac{1}{2} & 0 & -1 & -\frac{1}{2} & 0\\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$	$L_D(\mathcal{G}') = \begin{bmatrix} 1 & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{2} & 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0$

The eigenvalues of $L(\mathcal{G})$ are $\{0, 0, 1, 3, 4\}$. Node 5 is isolated, so there are two components and the null eigenvalue has multiplicity 2. The eigenvalues of $L(\mathcal{G}')$ are $\{0, 0, 0, 1.382, 3.618\}$. One more null eigenvalue appears, since there is no path between node 2 and node 4, and viceversa. The eigenvalues of $L_D(\mathcal{G})$ are $\{0, 0, 0.77, 1.5, 1.73\}$ and the ones of $L_D(\mathcal{G}')$ are $\{0, 0, 0, 0.59, 1.41\}$, all lying in a disk of radius 1 centered at the point 1 + 0j in the complex plane.

AUV Dynamics

In this chapter we derive the equations that rule the dynamics of an underwater vehicle. The coordinate frames required to describe the vehicle motion are defined in Section 3.1. The AUV's six degrees of freedom kinematic and dynamic equations are then analysed (Section 3.2), to obtain a simplified model for motion on the horizontal plane. A description of the Sirene AUV (Section 3.3) concludes the chapter. All the images are borrowed from (Aguiar, 1996).

3.1 Coordinate frames

To derive the equations of motion of an underwater vehicle in six degrees of freedom (DOF) it is standard practice to define an earth-fixed inertial frame $\{\mathcal{U}\}$, composed of the orthonormal axes $\{\boldsymbol{x}_U, \boldsymbol{y}_U, \boldsymbol{z}_U\}$ and a body-fixed frame $\{\mathcal{B}\}$, composed of the orthonormal axes $\{\boldsymbol{x}_B, \boldsymbol{y}_B, \boldsymbol{z}_B\}$, as indicated in Fig. 3.1. The body axes, two of which coincide with principal axes of inertia of the vehicle, are defined as follows (Fossen, 1994*a*):

- x_B is the longitudinal axis (directed from aft to fore),
- y_B is the transverse axis (directed from port to starboard),
- z_B is the normal axis (directed from top to bottom).

To simplify the model equations the origin O of the body-fixed frame is usually chosen to coincide with the center of gravity (CG) of the vehicle. The position and orientation of the vehicle are described with respect to the inertial reference frame $\{\mathcal{U}\}$. The linear and angular velocities of the vehicle, although measured with respect to the same inertial frame, are expressed in the body-fixed coordinate system $\{\mathcal{B}\}$. The geometry of the motion is analyzed in terms of Euler angles. With the frame definitions in Fig. 3.1 the following entities are defined by adopting the SNAME¹ notation (Fossen, 1994*a*):

• $\boldsymbol{\eta}_1 = [x, y, z]^T$ - position of the origin of $\{\mathcal{B}\}$ expressed in $\{\mathcal{U}\}$;

¹Society of Naval Architectures and Marine Engineers



Figure 3.1: Body-fixed and Earth-fixed reference frames

- $\eta_2 = [\phi, \theta, \psi]^T$ angles of roll, pitch and yaw that parametrize locally the orientation of $\{\mathcal{B}\}$ with respect to $\{\mathcal{U}\}$;
- $\boldsymbol{\nu}_1 = [u, v, w]^T$ linear velocities (surge, sway and heave) of the origin of $\{\mathcal{B}\}$ relative to $\{\mathcal{U}\}$, expressed in $\{\mathcal{B}\}$;
- $\boldsymbol{\nu}_2 = [p, q, r]^T$ angular velocity of $\{\mathcal{B}\}$ relative to $\{\mathcal{U}\}$ and expressed in $\{\mathcal{B}\}$;
- $\boldsymbol{\tau}_1 = [X, Y, Z]^T$ actuating forces expressed in $\{\mathcal{B}\};$
- $\boldsymbol{\tau}_2 = [K, M, N]^T$ actuating torques expressed in $\{\mathcal{B}\}$.

These entities can be written in a compact form to obtain the augmented vectors $\boldsymbol{\eta} = [\boldsymbol{\eta}_1^T, \boldsymbol{\eta}_2^T]^T$, $\boldsymbol{\nu} = [\boldsymbol{\nu}_1^T, \boldsymbol{\nu}_2^T]^T$ and $\boldsymbol{\tau} = [\boldsymbol{\tau}_1^T, \boldsymbol{\tau}_2^T]^T$.

3.2 Equations of motion

This section describes the kinematic and dynamic equations of motion of an autonomous underwater vehicle. The analysis of the general model of AUV is based mainly on (Fossen, 1994*b*) and (Balchen and Yin, 1994), while the more specific considerations about the Sirene vehicle borrow considerably from the work in (Aguiar, 1996; Rodrigues, 1997; Aguiar, 2002).

3.2.1 Kinematic equations

With the notation introduced in Section 3.1 the kinematic equations can be expressed in a compact form as

$$\begin{bmatrix} \dot{\boldsymbol{\eta}}_1 \\ \dot{\boldsymbol{\eta}}_2 \end{bmatrix} = \begin{bmatrix} J_1(\boldsymbol{\eta}_2) & \mathbf{0} \\ \mathbf{0} & J_2(\boldsymbol{\eta}_2) \end{bmatrix} = \begin{bmatrix} \boldsymbol{\nu}_1 \\ \boldsymbol{\nu}_2 \end{bmatrix} \quad \Longleftrightarrow \quad \dot{\boldsymbol{\eta}} = J(\boldsymbol{\eta})\boldsymbol{\nu}$$

where

$$J_1(\boldsymbol{\eta}_2) = \begin{bmatrix} c\psi c\theta & -s\psi c\theta + c\psi s\theta s\phi & s\psi s\phi + c\psi s\theta c\phi \\ s\psi c\theta & c\psi c\theta + s\psi s\theta s\phi & -c\psi s\phi + s\psi s\theta c\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix}$$

is the transformation matrix from $\{\mathcal{B}\}$ to $\{\mathcal{U}\}$, defined by means of three successive rotations (xyz convention), and

$$J_2(\boldsymbol{\eta}_2) = egin{bmatrix} 1 & s\phi t heta & c\phi t heta \ 0 & c\phi & -s\phi \ 0 & s\phi/c heta & c\phi/c heta \end{bmatrix}$$

is the matrix that relates body-fixed angular velocity with roll, pitch, and yaw rates. $J_2(\boldsymbol{\eta}_2)$ is undefined for a pitch angle $\theta = \pm \frac{\pi}{2}$, a consequence of using Euler angles to describe the vehicle's motion. This representation however remains adequate, since because of physical restrictions the AUV will always operate far from the singular point.

3.2.2 Dynamic equations

For underwater vehicles it is convenient to formulate Newton's laws in a local body-fixed coordinate system, so that the inertia tensor is constant in time and the hydrodinamic forces and moments, which are generated by the relative motion between the body and the fluid, can be expressed is a simple form. The rigid-body equations of motion can be expressed in compact form as

$$M_{RB}(\dot{\boldsymbol{\nu}}) + C_{RB}(\boldsymbol{\nu})\boldsymbol{\nu} = \boldsymbol{\tau}_{RB}$$
(3.1)

where τ_{RB} is a generalized vector of external forces and moments, M_{RB} is the rigid-body inertia matrix, constant, symmetrical and positive definite, and $C_{RB}(\nu)$ is the rigid-body Coriolis and centripetal matrix, parametrized so to be skew-symmetrical:

$$M_{RB} = 0$$

$$M_{RB} = M_{RB}^T > 0$$

$$C_{RB}(\boldsymbol{\nu}) = -C_{RB}^T(\boldsymbol{\nu}), \quad \forall \ \boldsymbol{\nu} \in \mathbb{R}$$

The extended expressions of M_{RB} and $C_{RB}(\nu)$ can be found in (Balchen and Yin, 1994; Aguiar, 1996; Rodrigues, 1997). Assuming that the contribution from sea currents and waves can be neglected τ_{RB} can be decomposed as

$$\boldsymbol{\tau}_{RB} = \boldsymbol{\tau} - \boldsymbol{\tau}_A - D(\boldsymbol{\nu})\boldsymbol{\nu} - \boldsymbol{g}(\boldsymbol{\eta}) \tag{3.2}$$

au is the vector of the thruster forces.

 τ_A is the hydrodynamic added mass term, a vector of pressure-induced forces and moments caused by a forced harmonic motion of the body that can be modeled as an increase of the body's inertia. Added mass forces and moments can be separated in terms which belong to an added inertia matrix M_A and a hydrodynamic Coriolis and centripetal matrix $C_A(\nu)$. For a completely submerged vehicle under the assumption of ideal fluid, no incident waves, no sea currents and frequency independence the added inertia matrix is positive definite and constant; moreover in many practical operations it can be assumed to be diagonal. The hydrodynamic Coriolis and centripetal matrix can be parametrized to be skew-symmetrical:

$$\begin{split} \dot{M}_A &= 0 \\ M_A &= M_A^T > 0 \\ C_A(\boldsymbol{\nu}) &= -C_A^T(\boldsymbol{\nu}), \quad \forall \; \boldsymbol{\nu} \in \mathbb{R} \end{split}$$

See (Balchen and Yin, 1994; Aguiar, 1996; Rodrigues, 1997) for the extended expressions of M_A and $C_A(\nu)$.

 $D(\boldsymbol{\nu})$, which contains terms related to hydrodynamic damping and lift forces due to viscous effects, is positive definite.

 $g(\eta)$ are the weight and buoyancy forces and moments due to gravity and fluid density, transformed to the body-fixed reference frame. They are also known as *restoring* forces because when the distance between the center of buoyancy and the center of mass, called *metacentric height*, is positive, *i.e.* the center of buoyancy is above the center of mass, they generate a stabilizing moment around the pitch axis. Combining (3.1) and (3.2) the 6 DOF body-fixed dynamic equations can be expressed in compact form as:

$$M\dot{\boldsymbol{\nu}} + C(\boldsymbol{\nu})\boldsymbol{\nu} + D(\boldsymbol{\nu})\boldsymbol{\nu} = \boldsymbol{\tau}$$

where

$$M = M_{RB} + M_A$$
$$C(\boldsymbol{\nu}) = C_{RB}(\boldsymbol{\nu}) + C_A(\boldsymbol{\nu}).$$

3.2.3 Simplified equations of motion

Assumptions can be made under which the vehicle will maintain its motion in the horizontal plane (Fossen, 1994a). With these constraints, the kinematic equations of motion take the



Figure 3.2: Body-fixed frame and inertial frame on the horizontal plane

form (see Fig. 3.2)

$$\dot{x} = u\cos(\psi) - v\sin(\psi) \tag{3.3a}$$

$$\dot{y} = u\sin(\psi) + v\cos(\psi) \tag{3.3b}$$

$$\dot{\psi} = r$$
 (3.3c)

Introducing a position vector $\boldsymbol{p} = [x, y]^T$, a linear velocity vector $\boldsymbol{v} = [u, v]^T$ and an orthonormal transformation matrix $R(\psi)$ from $\{\mathcal{B}\}$ to $\{\mathcal{U}\}$ for the simplified 3 DOF model, (3.3a) and (3.3b) can be written in compact form as

$$\dot{\boldsymbol{p}} = R(\psi)\boldsymbol{v} \tag{3.4}$$

The starboard and port thruster forces on the horizontal plane are denoted respectively by F_s and F_p , and l is their moment arm with respect to the center of geometry and mass of the AUV, which are assumed to coincide. No side thruster is present, since the vehicle is underactuated. The control inputs are

$$\tau_u = F_s + F_p$$

$$\tau_r = l(F_s - F_p)$$

respectively the pushing force along the vehicle axis x_B and the steering torque about its vertical axis z_B . Assuming that the AUV is neutrally buoyant and that the centre of buoyancy coincides with the centre of mass the dynamic equations of motion for surge, sway and heading yield

$$m_u \dot{u} - m_v vr + d_u u = \tau_u \tag{3.5a}$$

$$m_v \dot{v} + m_u ur + d_v v = 0 \tag{3.5b}$$

$$m_r \dot{r} - m_{uv} uv + d_r r = \tau_r \tag{3.5c}$$

where

$$\begin{split} m_u &= m - X_{\dot{u}}, & d_u &= -X_u - X_{|u|u}|u|, \\ m_v &= m - Y_{\dot{v}}, & d_v &= -Y_v - Y_{|v|v}|v|, \\ m_r &= I_z - N_{\dot{r}}, & d_r &= -N_r - N_{|r|r}|r|, \\ m_{uv} &= m_u - m_v, \end{split}$$

In presence of a constant irrotational ocean current v_c forming an angle ϕ_c with respect to the fixed frame, the kinematic equations (3.3) hold, with $u = u_r + u_c$ and $v = v_r + v_c$, where u_r and v_r are the components of the velocity of the vehicle with respect to the current, and u_c and v_c are the components of the ocean current velocity in the body-frame reference. In this case the dynamic equations (3.5) must be modified to

$$m_u \dot{u}_r - m_v v_r r + d_{u_r} u_r = \tau_u \tag{3.6a}$$

$$m_v \dot{v}_r + m_u u_r r + d_{v_r} v_r = 0 ag{3.6b}$$

$$m_r \dot{r} - m_{uv} u_r v_r + d_r r = \tau_r \tag{3.6c}$$

where $d_{u_r} = -X_u - X_{|u|u|} |u_r|$ and $d_{v_r} = -Y_v - Y_{|v|v|} |v_r|$. The equations of the actuated dynamics can be written in a compact form as

$$M\dot{\boldsymbol{u}} + C(\boldsymbol{v}_r)\boldsymbol{u} + D\boldsymbol{u} = \boldsymbol{\tau} \tag{3.7}$$

where $\boldsymbol{u} = [u_r, r]^T$ and

$$M = \begin{bmatrix} m_u & 0\\ 0 & m_r \end{bmatrix}$$
$$C = \begin{bmatrix} 0 & -m_v v_r\\ -m_{uv} v_r & 0 \end{bmatrix}$$
$$D = \begin{bmatrix} d_{u_r} & 0\\ 0 & d_r \end{bmatrix}$$

Notice that M and D are positive definite.



Figure 3.3: The Sirene AUV with benthic lab

When a vehicle is underactuated, the unactuated dynamics imply constraints on the accelerations. It is demonstrated in (Aguiar, 2002) that the constraint represented by equation (3.6b) is not integrable, meaning that the system is second order nonholonomic. In contrast to the first order nonholonomic case (Bloch *et al.*, 1992), such a constraint does not reduce the dimension of the state space, so a set of three independent configuration variables and three velocity variables is required to completely specify the state of the AUV in the horizontal plane.

3.3 The Sirene

One of the techniques employed in the exploration of the oceans consists in placing on the seabed *benthic stations*, platforms equipped with dedicated instrumentation, especially designed for the purpose of gathering measures and samples. The Sirene (Figures 3.3 and 3.4) is an underwater shuttle designed in the scope of the MAST-II European project DESIBEL (New Methods for Deep Sea Intervention on Future Benthic Laboratories), coordinated by the French agency IFREMER², to automatically position benthic labs on the seabed down to depths of 4000 m. A typical mission is illustrated in Fig. 3.5. With no benthic lab the Sirene is 4.0 meters long, 1.6 meters wide and 1.96 meters high, and it weighs four tons. It is equipped with two back thrusters (2 kW) which control surge and yaw motion in the horizontal plane, while a vertical thruster (1.2 kW) regulates depth. Roll and pitch motion are not controlled since the metacentric height is sufficiently large (36 cm) to ensure static

²French Research Institute for Exploitation of the Sea





Legend:

- ${\ensuremath{\mathfrak{O}}}$ Center of mass (Sirene)
- \circledast Center of buoyancy (Sirene)
- ${\ensuremath{\mathfrak{O}}}$ Center of mass (Sirene + Lab)
- $\textcircled{\black}$ Center of buoyancy (Sirene + Lab)

Figure 3.4: Rear view, left side view and top view of the Sirene with benthic lab

$$\begin{split} m &= 4000 \mbox{ kg } & X_{\dot{u}} = -290 \mbox{ kg } & X_u = -360 \mbox{ kg/s } & X_{|u|u} = -805 \mbox{ kg/m} \\ I_z &= 2660 \mbox{ kg } m^2 \ Y_{\dot{v}} = -310 \mbox{ kg } & Y_v = -420 \mbox{ kg/s } & Y_{|v|v} = -1930 \mbox{ kg/m} \\ & N_{\dot{r}} = -95 \mbox{ kg } m^2 \ N_r = -110 \mbox{ kg } m/s \ N_{|r|r} = -555 \mbox{ kg } m \end{split}$$

Table 3.1: Parameters of the simplified model of the Sirene AUV



Figure 3.5: Mission scenario: installing a benthic lab on the seabed

stability. A detailed description of the model parameters obtained from a series of tests on a quarter scale model of the Sirene can be found in (Aguiar, 1996; Aguiar and Pascoal, 1997). The parameters of the simplified model are shown in Table 3.1.

AUV Motion Control

In this chapter the tracking and path-following problems are solved by decoupling them into a dynamic and a kinematic task. The controllers for the inner and the outer loop are designed independently in Section 4.2 and Section 4.4. The stability and convergence of the closed-loop system are then formally proved in Section 4.6.

4.1 Introduction

Trajectory tracking problems are concerned with the design of control laws that force a vehicle to reach and follow a time parameterized reference *i.e.*, a geometric path with an associated timing law (Aguiar and Hespanha, 2003). In path-following problems a vehicle is required to converge to and follow a path that is specified without a temporal law (Aguiar and Hespanha, 2004). Once in the path, the vehicle should follow it with a desired speed profile. Typically, smoother convergence to a path is achieved in this case when compared with the performance obtained with trajectory tracking controllers, and the control signals are less likely pushed to saturation (Pascoal et al., 2005). Moreover, in (Aguiar et al., 2005, 2007a), it is proved that in path-following, the performance limitations due to unstable zerodynamics can be removed. A possible solution to the path-following problem, which was adopted for the control of wheeled robots in (Micaelli and Samson, 1993) and of underwater vehicles in (Encarnação and Pascoal, 2000), is to design a controller that computes i) the distance between the vehicle's center of mass and the closest point P on the path and ii) the angle between the vehicle's total velocity vector \boldsymbol{v} and the tangent to the path at P, and reduces both to zero. This is usually done by introducing a Serret-Frenet frame $\{\mathcal{F}\}\$ that moves along the path and plays the role of the body axis of a virtual target vehicle that should be tracked by the real vehicle. This approach however poses stringent initial condition constraints and introduces some singularities. The solution proposed in (Aicardi et al., 2001; Soetanto et al., 2003; Lapierre et al., 2003b) to lift these limitations is to parameterize the geometric path by a variable γ and control explicitly the rate of progression $\dot{\gamma}$ of the virtual target. This design procedure, followed in this thesis, effectively creates an extra degree of freedom and allows to bypass the problems that arise when the position of the virtual target is simply defined by the projection of the actual vehicle



Figure 4.1: The motion control problem is divided into a dynamic and a kinematic task

on that path. Both trajectory tracking and path-following problems can be tackled by decomposing the motion-control problem into an inner-loop dynamic task, which consists of making the vehicle move at a desired speed, and an outer-loop kinematic task, which assigns the reference speed so to achieve convergence to the path (see Fig. 4.1), with or without the constraint of a timing law. While better results could be achieved, in terms of saturation and smoothness of the control signal, with a single control law for the vehicle thrusters, based on a full state feedback, dividing the problem results in simpler control laws. Moreover, it is common for AUVs to be equipped with inner-loop velocity controllers that regulate the thrust. Therefore, the same kinematic outer-loop controller could be mounted on a wide range of AUVs, regardless of the parameters that define their dynamics.

4.2 Dynamic controller

Problem 4.1 (Inner loop). Consider the underwater autonomous vehicle with dynamic equations given by (3.6). Let $\mathbf{u}_d(t) = [u_d, r_d]^T \in \mathbb{R}^2$ be a desired speed assignment and suppose that \mathbf{u}_d is sufficiently smooth and its time derivative is bounded. Derive a feedback control law for $\boldsymbol{\tau} = [\tau_u, \tau_r]^T$ such that $\mathbf{u} = [u_r, r]^T$ converges to \mathbf{u}_d exponentially fast.

Define the error between the actual and desired velocities

$$\tilde{\boldsymbol{u}}(t) = \boldsymbol{u}(t) - \boldsymbol{u}_d(t)$$

Convergence of \boldsymbol{u} to \boldsymbol{u}_d is equivalent to convergence of $\tilde{\boldsymbol{u}}$ to the origin. The actuated dynamics equation (3.7) can be rewritten introducing the error $\tilde{\boldsymbol{u}}$:

$$M\dot{\tilde{\boldsymbol{u}}} = -C(v_r)\boldsymbol{u} - D\boldsymbol{u} - M\dot{\boldsymbol{u}}_d + \boldsymbol{\tau}$$

= $-C(v_r)\boldsymbol{u} - D\tilde{\boldsymbol{u}} - D\boldsymbol{u}_d - M\dot{\boldsymbol{u}}_d + \boldsymbol{\tau}$ (4.1)

Proposition 4.1. Consider the system described by (4.1) in closed-loop with the control

law

$$\boldsymbol{\tau} = -K_d(\boldsymbol{u} - \boldsymbol{u}_d) + M\dot{\boldsymbol{u}}_d + C(v_r)\boldsymbol{u} + D\boldsymbol{u}_d$$
(4.2)

where

$$K_d = \begin{bmatrix} k_u & 0\\ 0 & k_r \end{bmatrix}$$

is positive definite. The origin $\tilde{u} = 0$ is a globally exponentially stable equilibrium point for this system.

Proof. Substituting the control law (4.2) in (4.1) yields

$$\dot{\tilde{\boldsymbol{u}}} = -M^{-1}[K_d + D]\tilde{\boldsymbol{u}}$$
(4.3)

This equation describes the dynamics of the error in closed-loop. It is immediate to observe that $\tilde{u} = 0$ is an equilibrium point. Consider the following candidate Lyapunov function

$$V_d = \frac{1}{2} \tilde{\boldsymbol{u}}^T \tilde{\boldsymbol{u}} \tag{4.4}$$

and define $A = M^{-1}[K_d + D]$, symmetric and positive definite. The derivative of V_d with respect to time is

$$\dot{V}_d = \tilde{\boldsymbol{u}}^T \dot{\tilde{\boldsymbol{u}}} = -\tilde{\boldsymbol{u}}^T A \tilde{\boldsymbol{u}}$$

Then,

$$\frac{1}{2} \|\tilde{\boldsymbol{u}}\|^2 \le V_d \le \frac{1}{2} \|\tilde{\boldsymbol{u}}\|^2$$
$$\frac{\partial V_d}{\partial t} + \frac{\partial V_d}{\partial \tilde{\boldsymbol{u}}} f(t, \tilde{\boldsymbol{u}}) \le - \|A^{-1}\|^{-1} \|\tilde{\boldsymbol{u}}\|^2$$
(4.5)

All the conditions of Theorem 2.10 are satisfied and the origin is a globally exponentially stable equilibrium point of system (4.3), *i.e.* the speed error decays to zero exponentially with time. \Box

4.3 Ocean current observer

In the presence of ocean currents the dynamic control laws derived in Section 4.2 hold, since the inner control loop regulates the vehicle-current relative velocities. In the design of the kinematic control however the intensity of water current has to be taken into account, since it modifies the total velocity at which the vehicle is moving. There is no easy way of sensing the currents directly. Instead, let $v_{c_{xy}}$ denote the ocean current expressed in the



Figure 4.2: Ocean current observer

inertial reference frame $\{\mathcal{U}\}$, so that

$$\boldsymbol{v}_{c_{xy}} = R(\psi) \begin{bmatrix} u_c \\ v_c \end{bmatrix}$$

The kinematic equations (3.4) can be rewritten as

$$\dot{\boldsymbol{p}} = R(\psi) \boldsymbol{v}_r + \boldsymbol{v}_{c_{xy}}$$

A simple observer for the current $v_{c_{xy}}$, with the structure shown in Fig. 4.2 is (Aguiar and Pascoal, 2007b)

$$\dot{\hat{\boldsymbol{p}}} = R(\psi)\boldsymbol{v}_r + \hat{\boldsymbol{v}}_{cxy} + K_{p_{obs}}\tilde{\boldsymbol{p}}$$
(4.6a)

$$\dot{\boldsymbol{v}}_{c_{xy}} = K_{c_{obs}} \tilde{\boldsymbol{p}} \tag{4.6b}$$

where $K_{p_{obs}}$ and $K_{c_{obs}}$ are the observer gain diagonal matrices, and

$$oldsymbol{p} = oldsymbol{p} - oldsymbol{\hat{p}} \ ilde{oldsymbol{v}}_{c_{xy}} = oldsymbol{v}_{c_{xy}} - oldsymbol{\hat{v}}_{c_{xy}}$$

are the estimation errors. Assuming that $\dot{\boldsymbol{v}}_{c_{xy}} = 0$, that is, the current is constant, the error dynamics are described by

$$\tilde{\boldsymbol{p}} = -K_{p_{obs}}\tilde{\boldsymbol{p}} + \tilde{\boldsymbol{v}}_{c_{xy}}$$

 $\dot{\tilde{\boldsymbol{v}}}_{c_{xy}} = -K_{c_{obs}}\tilde{\boldsymbol{p}}$

If $K_{p_{obs}}$ and $K_{c_{obs}}$ are chosen so to be stricly positive the estimation errors have a globally exponentially stable equilibrium point in the origin, *i.e.*, \tilde{p} and $\tilde{v}_{c_{xy}}$ converge to zero exponentially fast. The symbol \hat{v}_c , with no reference frame index, will be used in the remainder of this thesis to denote the estimated water current velocity expressed in the body-fixed



Figure 4.3: Kinematic problem on the horizontal plane

reference frame $\{\mathcal{B}\}$:

$$\hat{\boldsymbol{v}}_c = R^T(\psi) \hat{\boldsymbol{v}}_{c_{xy}}$$

4.4 Kinematic controller

4.4.1 Trajectory tracking

To solve the problem of driving a vehicle along a time parametrized desired trajectory, the idea is to make the vehicle approach a virtual target that moves along the path with a defined timing law. Let $p_d(t)$ be the position of the target. The trajectory tracking problem for the outer loop can then be formulated as follows:

Problem 4.2. Consider the underwater autonomous vehicle with kinematic equations given by (3.3), and let $\mathbf{p}_d(t) : [0, \infty) \to \mathbb{R}^2$ be a continuously differentiable bounded time-varying desired trajectory. Derive a feedback control law for \mathbf{u} such that the position of the vehicle converges to and remains inside a tube, centered around the desired path, that can be made arbitrarily thin, i.e., $\|\mathbf{p}(t) - \mathbf{p}_d(\gamma(t))\|$ converges to a neighborhood of the origin that can be made arbitrarily small.

Referring to Fig. 4.3, define the position error expressed in body-frame coordinates

$$\boldsymbol{e} = R^T(\psi)(\boldsymbol{p}(t) - \boldsymbol{p}_d(t)) \tag{4.7}$$

Its dynamics are described by the expression

$$\dot{\boldsymbol{e}} = \dot{\boldsymbol{R}}^{T}(\psi)(\boldsymbol{p}(t) - \boldsymbol{p}_{d}(t)) + \boldsymbol{R}^{T}(\psi)(\dot{\boldsymbol{p}}(t) - \dot{\boldsymbol{p}}_{d}(t))$$
(4.8)

The time derivative of the rotation matrix is

$$\dot{R}^T(\psi) = -S(r)R^T(\psi)$$

where

$$S = \begin{bmatrix} 0 & -r \\ r & 0 \end{bmatrix}$$

Then,

$$\dot{\boldsymbol{e}}(t) = -S(r)R^{T}(\psi)(\boldsymbol{p}(t) - \boldsymbol{p}_{d}(t)) + R^{T}(\psi)(\dot{\boldsymbol{p}}(t) - \dot{\boldsymbol{p}}_{d}(t))$$

Substituting (4.7) and (3.4) into the foregoing expression yields

$$\dot{\boldsymbol{e}} = -S(r)\boldsymbol{e} + R^{T}(\psi)(R(\psi)\boldsymbol{v} - \dot{\boldsymbol{p}}_{d}(t))$$

$$= -S(r)(\boldsymbol{e} - \boldsymbol{\delta}) - S(r)\boldsymbol{\delta} + \boldsymbol{v} - R^{T}(\psi)\dot{\boldsymbol{p}}_{d}(t)$$
(4.9)

where the vector $\boldsymbol{\delta} = [\delta, 0]^T$, with δ being an arbitrarily small negative constant, has been introduced. As shown in Fig. 4.3, the new error vector $(\boldsymbol{e} - \boldsymbol{\delta})$ is the distance between the AUV's position and a neighborhood of the virtual target's position. The velocity vector \boldsymbol{v} is the sum of the velocity of the vehicle with respect to the current and of the velocity of the ocean current in the body-frame reference, *i.e.*, $\boldsymbol{v} = \boldsymbol{v}_r + \boldsymbol{v}_c$. In a fully actuated vehicle it would be possible to control both elements of the velocity vector \boldsymbol{v}_r and to choose expressions of u_r and v_r that would make the error converge to zero. However, as stated in Section 4.2, the control variable for the kinematic outer loop is $\boldsymbol{u} = [u_r, r]^T$. Equation (4.9) can be rearranged as

$$\dot{\boldsymbol{e}} = -S(r)(\boldsymbol{e} - \boldsymbol{\delta}) + \begin{bmatrix} 0 & r \\ -r & 0 \end{bmatrix} \begin{bmatrix} \delta \\ 0 \end{bmatrix} + \begin{bmatrix} u_r \\ v_r \end{bmatrix} + \begin{bmatrix} u_c \\ v_c \end{bmatrix} - R^T(\psi)\dot{\boldsymbol{p}}_d(t)$$

$$= -S(r)(\boldsymbol{e} - \boldsymbol{\delta}) + \begin{bmatrix} u_r \\ -r\delta \end{bmatrix} + \begin{bmatrix} 0 \\ v_r \end{bmatrix} + \begin{bmatrix} u_c \\ v_c \end{bmatrix} - R^T(\psi)\dot{\boldsymbol{p}}_d(t)$$

$$= -S(r)(\boldsymbol{e} - \boldsymbol{\delta}) + \Delta \boldsymbol{u} + \begin{bmatrix} 0 \\ v_r \end{bmatrix} + \boldsymbol{v}_c - R^T(\psi)\dot{\boldsymbol{p}}_d(t)$$
(4.10)

where

$$\Delta = \begin{bmatrix} 1 & 0 \\ 0 & -\delta \end{bmatrix}$$

It is now evident why the error is made to converge to a neighborhood of the origin, instead of to the origin itself: had not δ been introduced the control variable r would not appear in the position error dynamics.

Proposition 4.2. Consider the system described by (3.4) in closed-loop with the control

law

$$\boldsymbol{u} = \Delta^{-1} \left(-K_k \tanh(\boldsymbol{e} - \boldsymbol{\delta}) - \boldsymbol{v}_c - \begin{bmatrix} 0\\ v_r \end{bmatrix} + R^T(\psi) \dot{\boldsymbol{p}}_d(t) \right)$$
(4.11)

where

$$K_k = \begin{bmatrix} k_x & 0\\ 0 & k_y \end{bmatrix}$$

is positive definite. Then $e = \delta$ is a globally asymptotically stable equilibrium point for this system.

Proof. Define the candidate Lyapunov function

$$V_k = \frac{1}{2} (\boldsymbol{e} - \boldsymbol{\delta})^T (\boldsymbol{e} - \boldsymbol{\delta})$$
(4.12)

whose time derivative is

$$\dot{V}_k = (\boldsymbol{e} - \boldsymbol{\delta})^T \dot{\boldsymbol{e}} \tag{4.13}$$

Substituting the control law (4.11) in (4.10) yields

$$\dot{\boldsymbol{e}} = -S(r)(\boldsymbol{e} - \boldsymbol{\delta}) - K_k \tanh(\boldsymbol{e} - \boldsymbol{\delta})$$
(4.14)

The foregoing expression describes a system for which $e = \delta$ is an equilibrium point. Substituting (4.14) into (4.13), and remembering that S(r) is skew-symmetric

$$\dot{V}_k = -(\boldsymbol{e} - \boldsymbol{\delta})^T S(r)(\boldsymbol{e} - \boldsymbol{\delta}) - (\boldsymbol{e} - \boldsymbol{\delta})^T K_k \tanh(\boldsymbol{e} - \boldsymbol{\delta})$$
$$= -(\boldsymbol{e} - \boldsymbol{\delta})^T K_k \tanh(\boldsymbol{e} - \boldsymbol{\delta})$$

Since K_k is symmetric and positive definite, we can conclude that

$$\frac{1}{2} \|\boldsymbol{e} - \boldsymbol{\delta}\|^2 \le V_k \le \frac{1}{2} \|\boldsymbol{e} - \boldsymbol{\delta}\|^2$$
(4.15)

$$\frac{\partial V_k}{\partial t} + \frac{\partial V_k}{\partial \boldsymbol{e}} f(t, \boldsymbol{e}) \le -\tanh(\boldsymbol{e} - \boldsymbol{\delta})^T K_k \tanh(\boldsymbol{e} - \boldsymbol{\delta}) \le 0$$
(4.16)

The function $\frac{1}{2} \| \boldsymbol{e} - \boldsymbol{\delta} \|^2$ is radially unbounded. Hence, all the conditions of Theorem 2.9 are satisfied and $\boldsymbol{e} = \boldsymbol{\delta}$, where $\boldsymbol{\delta}$ can be chosen arbitrarily close to the origin, is a globally asymptotically stable equilibrium point.

Remark. Observe from (4.11) that choosing a smaller value of δ , so that the equilibrium point is closer to the origin, corresponds to an increase in the intensity of the control input r. In view of this, a possible alternative control strategy is to view δ as an additional input signal and shape its behavior accordingly to the value of e and the maximum input r. This requires further investigation and is not pursued here.



Figure 4.4: Trajectory-tracking controller

In (4.11) it was assumed in the design of the tracking controller (Fig. 4.4) that the term $\dot{\boldsymbol{p}}_d(t)$ is known. This is immediate in applications where the path and the timing law are determined *a priori*. When instead only $\boldsymbol{p}_d(t)$, and not its time derivative, is accessible, as in the case of following a target whose position is detected through sensors, $\dot{\boldsymbol{p}}_d(t)$ may be obtained through filtering. See for example (Aguiar *et al.*, 2003) where experimental results are reported of using nonlinear filtering to make an hovercraft track a target.

4.4.2 Path-following (strategy I)

Lifting the temporal constraint deriving from the timing law, the path-following problem can be formulated as follows:

Problem 4.3. Consider the underwater autonomous vehicle with kinematic equations given by (3.3), and let $\mathbf{p}_d(\gamma) \in \mathbb{R}^2$ be a desired path parameterized by a continuous variable $\gamma(t) \in \mathbb{R}$ and $v_d(\gamma) \in \mathbb{R}$ a desired speed assignment. Suppose also that $\mathbf{p}_d(\gamma)$ is sufficiently smooth and its derivatives with respect to γ are bounded. Design a controller such that the position of the vehicle i) converges to and remains inside a tube, centered around the desired path, that can be made arbitrarily thin, i.e., $\|\mathbf{p}(t) - \mathbf{p}_d(\gamma(t))\|$ converges to a neighborhood of the origin that can be made arbitrarily small, and ii) satisfies a desired speed assignment v_d along the path, i.e., $|\dot{\gamma}(t) - v_d(\gamma(t))| \to 0$ as $t \to \infty$.

The position error (4.7) can be redefined for path-following as

$$\boldsymbol{e} = R^T(\boldsymbol{\psi})(\boldsymbol{p}(t) - \boldsymbol{p}_d(\boldsymbol{\gamma}(t))) \tag{4.17}$$

Then, expression (4.10), describing the error dynamics, becomes

$$\dot{\boldsymbol{e}} = -S(r)(\boldsymbol{e} - \boldsymbol{\delta}) + \Delta \boldsymbol{u} + \begin{bmatrix} 0\\v_r \end{bmatrix} + \boldsymbol{v}_c - R^T(\psi)\dot{\boldsymbol{p}}_d(\gamma)$$
$$= -S(r)(\boldsymbol{e} - \boldsymbol{\delta}) + \Delta \boldsymbol{u} + \begin{bmatrix} 0\\v_r \end{bmatrix} + \boldsymbol{v}_c - R^T(\psi)\frac{\partial \boldsymbol{p}_d(\gamma)}{\partial \gamma}\dot{\gamma}$$
(4.18)

where the dependence of the time derivative of the desired path from the parameter γ has been made explicit. Define the speed error

$$z = \dot{\gamma}(t) - v_d(\gamma(t)) \tag{4.19}$$

Note that the speed assignment v_d is not an actual velocity: it expresses the rate at which the parameter γ changes. The desired speed of the virtual target is $\frac{\partial \mathbf{p}_d(\gamma)}{\partial \gamma} v_d$. Imposing the equality

$$\dot{\gamma} = v_d(\gamma) \tag{4.20}$$

satisfies identically the speed assignment¹. Then, replacing the term $\dot{\boldsymbol{p}}_d(t)$ with $\frac{\partial \boldsymbol{p}_d(\gamma)}{\partial \gamma} v_d$ in the control law (4.11) brings to the same results of Proposition 4.2, that can be restated as follows.

Proposition 4.3. Consider the system described by (3.4) in closed-loop with the control law

$$\boldsymbol{u} = \Delta^{-1} \left(-K_k \tanh(\boldsymbol{e} - \boldsymbol{\delta}) - \boldsymbol{v}_c - \begin{bmatrix} 0\\ v_r \end{bmatrix} + R^T(\psi) \frac{\partial \boldsymbol{p}_d(\gamma)}{\partial \gamma} v_d \right)$$
(4.21)

where

$$K_k = \begin{bmatrix} k_x & 0\\ 0 & k_y \end{bmatrix}$$

is positive definite. Then $e = \delta$, arbitrarily close to the origin, is a globally asymptotically stable equilibrium point for this system, and the speed error z is identically null.

The path-following controller designed in this section is very similar to the trajectorytracking controller of Section 4.4.1. In fact, as it is shown in Fig. 4.5, the path-following controller can be seen as the cascade of a path-generator block and the trajectory-tracking controller (Fig. 4.4). The reason for this is that in both cases the virtual target progresses along the path with some velocity constraint, depending on time in the case of tracking, and on the position of the virtual target, *i.e.*, on the value of γ , in the case of path-following. However, as will be seen in Chapter 5, the introduction of a parameter γ is greatly relevant for two reasons. First, it is fundamental for the coordination of multiple vehicles. Second,

¹In Section 4.5 we show a different strategy to deal with the speed assignment constraint.



Figure 4.5: Path-following control (strategy I)

as

$$\gamma(t) = \gamma(t_0) + \int_{t_0}^t \dot{\gamma} dt$$

it introduces an additional flexibility to control the vehicle: the initial value $\gamma(t_0)$.

4.5 Path-following (strategy II)

An alternative control strategy can be adopted to solve the path-following problem without imposing the equality (4.20). Substituting (4.19) into (4.18) yields

$$\dot{\boldsymbol{e}} = -S(r)(\boldsymbol{e} - \boldsymbol{\delta}) + \Delta \boldsymbol{u} + \begin{bmatrix} 0\\v_r \end{bmatrix} + \boldsymbol{v}_c - R^T(\psi) \frac{\partial \boldsymbol{p}_d(\gamma)}{\partial \gamma}(z + v_d)$$
(4.22)

An expression for the speed error dynamics can be obtained from the time derivative of (4.19):

$$\dot{z} = \ddot{\gamma}(t) - \dot{v}_d(\gamma(t)) \tag{4.23}$$

Define a composite error vector $\boldsymbol{e}_c = [\boldsymbol{e} - \boldsymbol{\delta}, z]^T$, whose dynamics are described by

$$\dot{\boldsymbol{e}}_c = \begin{bmatrix} \dot{\boldsymbol{e}} \\ \dot{\boldsymbol{z}} \end{bmatrix} \tag{4.24}$$

The path-following problem can now be viewed as determining a control law for \boldsymbol{u} and $\ddot{\gamma}$ that drives \boldsymbol{e}_c to **0** (see Fig. 4.6). The additional control variable $\ddot{\gamma}$ was introduced through the technique known as *backstepping* (Khalil, 2002).

Proposition 4.4. Consider the system described by (3.4) in closed-loop with the control



Figure 4.6: Path-following control (strategy II)

laws

$$\boldsymbol{u} = \Delta^{-1} \left(-K_k \tanh(\boldsymbol{e} - \boldsymbol{\delta}) - \boldsymbol{v}_c - \begin{bmatrix} 0 \\ v_r \end{bmatrix} + R^T(\boldsymbol{\psi}) \frac{\partial \boldsymbol{p}_d(\boldsymbol{\gamma})}{\partial \boldsymbol{\gamma}} v_d \right)$$
(4.25a)

$$\ddot{\gamma} = -k_z z + \frac{\partial v_d(\gamma)}{\partial \gamma} \dot{\gamma} + (\boldsymbol{e} - \boldsymbol{\delta})^T R^T(\psi) \frac{\partial \boldsymbol{p}_d(\gamma)}{\partial \gamma}$$
(4.25b)

where k_z is a positive constant and

$$K_k = \begin{bmatrix} k_x & 0 \\ 0 & k_y \end{bmatrix}$$

is positive definite. The origin $e_c = 0$ is a globally asymptotically stable equilibrium point for this system.

Proof. Define a Lyapunov function for the position error as in (4.12), a second Lyapunov function for the speed error

$$V_z = \frac{1}{2}z^2$$

and a composite Lyapunov function

$$V_c = \frac{1}{2} \boldsymbol{e}_c^T \boldsymbol{e}_c = V_k + V_z$$

The time derivative of V_c is

$$\dot{V}_c = \dot{V}_k + \dot{V}_z = (\boldsymbol{e} - \boldsymbol{\delta})^T \dot{\boldsymbol{e}} + z^T \dot{z}$$
(4.26)

Substituting (4.22) and (4.23) into the foregoing expression yields

$$\dot{V}_{c} = (\boldsymbol{e} - \boldsymbol{\delta})^{T} \left(\Delta \boldsymbol{u} + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{v}_{r} \end{bmatrix} + \boldsymbol{v}_{c} - R^{T} \frac{\partial \boldsymbol{p}_{d}}{\partial \gamma} (\boldsymbol{z} + \boldsymbol{v}_{d}) \right) + \boldsymbol{z} (\ddot{\gamma} - \dot{\boldsymbol{v}}_{d})$$

$$= (\boldsymbol{e} - \boldsymbol{\delta})^{T} \left(\Delta \boldsymbol{u} + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{v}_{r} \end{bmatrix} + \boldsymbol{v}_{c} - R^{T} \frac{\partial \boldsymbol{p}_{d}}{\partial \gamma} \boldsymbol{v}_{d} \right)$$

$$+ \boldsymbol{z} \left(\ddot{\gamma} - \dot{\boldsymbol{v}}_{d} - (\boldsymbol{e} - \boldsymbol{\delta})^{T} R^{T} \frac{\partial \boldsymbol{p}_{d}}{\partial \gamma} \right)$$

$$(4.27)$$

Defining

$$K_c = \begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & k_z \end{bmatrix}$$

and substituting the control laws (4.25) in (4.27)

$$\dot{V}_{c} = -(\boldsymbol{e} - \boldsymbol{\delta})^{T} K_{k} \tanh(\boldsymbol{e} - \boldsymbol{\delta}) - k_{z} z^{2}$$

$$\leq -\boldsymbol{e}_{c}^{T} K_{c} \tanh \boldsymbol{e}_{c} \qquad (4.28)$$

The following inequalities are verified:

$$\frac{1}{2} \|\boldsymbol{e}_{c}\|^{2} \le V_{c} \le \frac{1}{2} \|\boldsymbol{e}_{c}\|^{2}$$
(4.29)

$$\frac{\partial V_c}{\partial t} + \frac{\partial V_c}{\partial \boldsymbol{e}_c} f(t, \boldsymbol{e}_c) \le -\boldsymbol{e}_c^T K_c \tanh \boldsymbol{e}_c \le 0$$
(4.30)

The function $\frac{1}{2} \|\boldsymbol{e}_{c}\|^{2}$ is radially unbounded. Hence, all the conditions of Theorem 2.9 are satisfied and the origin $\boldsymbol{e}_{c} = \mathbf{0}$ is a globally asymptotically stable equilibrium point: $\boldsymbol{e} = \boldsymbol{p} - \boldsymbol{p}_{d}$ and $z = \dot{\gamma} - v_{d}$ converge respectively to $\boldsymbol{\delta}$ and 0.

The difference with the path-following controller designed in Section 4.4.2 is that now the evolution of the position of the virtual target p_d also depends on the position error $(e - \delta)$. If the error is positive, that is, the AUV is behind the desired position, then the virtual target moves slower, to allow the vehicle to reach it. Viceversa, if the vehicle is in front of the virtual target, this moves faster. This is illustrated in Section 6.1.

4.6 Closed-loop controllers

4.6.1 Tracking controller

In the previous sections, control laws for the inner and outer loop have been designed independently. This however is not sufficient to guarantee that all closed-loop signals converge,



Figure 4.7: The dynamic and the kinematic controllers in closed-loop

or are bounded. In particular, the speed requirements produced by the outer-loop control are not satisfied identically, but are instead assigned to $u_d = u - \tilde{u}$, so the dynamics of the inner loop will affect the outer loop, and expression (4.10), describing the dynamics of the position error, becomes

$$\dot{\boldsymbol{e}} = -S(r)(\boldsymbol{e} - \boldsymbol{\delta}) + \Delta(\boldsymbol{u}_d + \tilde{\boldsymbol{u}}) + \begin{bmatrix} 0\\ v_r \end{bmatrix} + \boldsymbol{v}_c - R^T(\psi)\dot{\boldsymbol{p}}_d(t)$$
(4.31)

Furthermore, the control laws for both loops require for the sway velocity v_r to be known. The sensors that measure the sway velocity, however, are very expensive, so it was chosen to remove the terms containing v_r from the control laws, and view them as input perturbations that limit the performance of the system. The same applies to \dot{u}_d , which appears in the control law (4.2): if the two control loops are to be kept independent this term should not be available to the inner loop, as it is common for the inner-loop speed controllers mounted on AUVs to have only the reference velocities, and not their derivatives, as inputs. Therefore new control laws, not containing v_r nor \dot{u}_d , are adopted (see Fig. 4.7).

Theorem 4.1. Consider the system described by (3.4) and (3.6) in closed-loop with the ocean current observer (4.6) and the control laws

$$\boldsymbol{\tau} = -K_d(\boldsymbol{u} - \boldsymbol{u}_d) + D\boldsymbol{u}_d \tag{4.32a}$$

$$\boldsymbol{u}_{d} = \Delta^{-1} \left(-K_{k} \tanh(\boldsymbol{e} - \boldsymbol{\delta}) - \hat{\boldsymbol{v}}_{c} + R^{T}(\psi) \dot{\boldsymbol{p}}_{d}(t) \right)$$
(4.32b)

where

$$K_d = \begin{bmatrix} k_u & 0\\ 0 & k_r \end{bmatrix}, \qquad K_k = \begin{bmatrix} k_x & 0\\ 0 & k_y \end{bmatrix}$$

are positive definite, and let $\dot{\mathbf{p}}_d$, $\ddot{\mathbf{p}}_d$ and \mathbf{v}_c be bounded inputs. If the gains K_d and K_k are sufficiently large, then the system is ISS with restrictions on the initial states $\mathbf{e}(0)$ and $v_r(0)$.

Input-to-state stability Define a Lyapunov function

$$V = \frac{1}{2} (\tilde{\boldsymbol{u}}^T \tilde{\boldsymbol{u}} + \boldsymbol{e}^T \boldsymbol{e})$$

Its time derivative is

$$\dot{V} = \tilde{\boldsymbol{u}}^T \dot{\tilde{\boldsymbol{u}}} + \boldsymbol{e}^T \dot{\boldsymbol{e}}$$

Substituting the control laws (4.32) in (4.1) and (4.31) yields

$$\dot{\tilde{\boldsymbol{u}}} = -M^{-1} \left[K_d + D \right] \tilde{\boldsymbol{u}} - M^{-1} C(v_r) \boldsymbol{u} - \dot{\boldsymbol{u}}_d$$
(4.33a)

$$\dot{\boldsymbol{e}} = -S(r)(\boldsymbol{e} - \boldsymbol{\delta}) - K_k \tanh(\boldsymbol{e} - \boldsymbol{\delta}) + \Delta \tilde{\boldsymbol{u}} + [0, v_r]^T$$
(4.33b)

Then,

$$\dot{V} = -\tilde{\boldsymbol{u}}^T M^{-1} \left[K_d + D + C(v_r) \right] \tilde{\boldsymbol{u}} - \tilde{\boldsymbol{u}}^T M^{-1} C(v_r) \boldsymbol{u}_d - \tilde{\boldsymbol{u}}^T \dot{\boldsymbol{u}}_d - (\boldsymbol{e} - \boldsymbol{\delta})^T K_k \tanh(\boldsymbol{e} - \boldsymbol{\delta}) + (\boldsymbol{e} - \boldsymbol{\delta})^T \Delta \tilde{\boldsymbol{u}} + (\boldsymbol{e} - \boldsymbol{\delta})^T [0, v_r]^T \quad (4.34)$$

Remembering that $\boldsymbol{v}_c = R^T \boldsymbol{v}_{c_{xy}}$ The time derivative of the reference speed \boldsymbol{u}_d is

$$\dot{\boldsymbol{u}}_{d} = \Delta^{-1} \left(-4K_{k}B\dot{\boldsymbol{e}} + S(r)R^{T}\boldsymbol{v}_{c_{xy}} - S(r)R^{T}\dot{\boldsymbol{p}}_{d}(t) + R^{T}\ddot{\boldsymbol{p}}_{d}(t) \right)$$

= $-4K_{k}B\tilde{\boldsymbol{u}} + \boldsymbol{a}$ (4.35)

where

$$\boldsymbol{a} = \Delta^{-1} \left(4K_k B \left(S(r)(\boldsymbol{e} - \boldsymbol{\delta}) + K_k \tanh(\boldsymbol{e} - \boldsymbol{\delta}) + [0, v_r]^T \right) + S(r) R^T \boldsymbol{v}_{c_{xy}} - S(r) R^T \dot{\boldsymbol{p}}_d(t) + R^T(\psi) \ddot{\boldsymbol{p}}_d(t) \right) \quad (4.36)$$

and $B = \cosh^{-1}(\boldsymbol{e} - \boldsymbol{\delta}) \cosh^{-1}(\boldsymbol{e} - \boldsymbol{\delta})^T$. Substituting (4.35) in (4.34) yields

$$\dot{V} = -\tilde{\boldsymbol{u}}^T \left[M^{-1} \left[K_d + D + C(v_r) \right] - 4K_k B \right] \tilde{\boldsymbol{u}} - \tilde{\boldsymbol{u}}^T M^{-1} C(v_r) \boldsymbol{u}_d - \tilde{\boldsymbol{u}}^T \boldsymbol{a} - (\boldsymbol{e} - \boldsymbol{\delta})^T K_k \tanh(\boldsymbol{e} - \boldsymbol{\delta}) + (\boldsymbol{e} - \boldsymbol{\delta})^T \Delta \tilde{\boldsymbol{u}} + (\boldsymbol{e} - \boldsymbol{\delta})^T [0, v_r]^T \quad (4.37)$$

Applying Young's inequality

$$uw \le \frac{1}{\mu}u^p + \mu^{\frac{1}{p-1}}w^{\frac{p}{p-1}}, \ \forall \ u \ge 0, \ w \ge 0, \ \mu > 0, \ p > 1$$

with p = 2 to the term $(\boldsymbol{e} - \boldsymbol{\delta})^T \Delta \tilde{\boldsymbol{u}}$ allows to rewrite (4.37) as the inequality

$$\dot{V} \leq -\tilde{\boldsymbol{u}}^{T} \left(\left[M^{-1} \left[K_{d} + D + C(v_{r}) \right] - 4K_{k}B - \mu\Delta^{2} \right] \tilde{\boldsymbol{u}} - M^{-1}C(v_{r})\boldsymbol{u}_{d} - \boldsymbol{a} \right) - (\boldsymbol{e} - \boldsymbol{\delta})^{T} \left(K_{k} \tanh(\boldsymbol{e} - \boldsymbol{\delta}) + \frac{1}{\mu}(\boldsymbol{e} - \boldsymbol{\delta}) + [0, v_{r}]^{T} \right)$$
(4.38)

Denoting

$$K_u = M^{-1} [K_d + D + C(v_r)] - 4K_k B - \mu \Delta^2$$

we conclude from (4.38) that $\dot{V} \leq 0$ when

$$(1 - \theta_u + \theta_u)\tilde{\boldsymbol{u}}^T K_u \tilde{\boldsymbol{u}} \ge \tilde{\boldsymbol{u}}^T \left[M^{-1} C(v_r) \boldsymbol{u}_d + \boldsymbol{a} \right]$$
(4.39)

$$(1 - \theta_e + \theta_e)(\boldsymbol{e} - \boldsymbol{\delta})^T K_k \tanh(\boldsymbol{e} - \boldsymbol{\delta}) \ge (\boldsymbol{e} - \boldsymbol{\delta})^T \left(\frac{1}{\mu}(\boldsymbol{e} - \boldsymbol{\delta}) + \begin{bmatrix} 0\\v_r \end{bmatrix}\right)$$
(4.40)

where $0 < \theta_u, \theta_e < 1$. Assume now that v_r is small enough for K_u to be positive definite. Then, when

$$\|\tilde{\boldsymbol{u}}\| \ge \frac{1}{\theta_u} \|K_u^{-1}\| \|M^{-1}C(v_r)\boldsymbol{u}_d + \boldsymbol{a}\|$$
(4.41)

$$\|\boldsymbol{e} - \boldsymbol{\delta}\| \ge \operatorname{arctanh}\left(\frac{1}{\theta_e} \left(\frac{1}{\mu} \|K_k^{-1}\| \|\boldsymbol{e} - \boldsymbol{\delta}\| + \frac{1}{k_r} |v_r|\right)\right)$$
(4.42)

it follows from (4.38), (4.39) and (4.40), that

$$\dot{V} \leq -(1-\theta_u)\tilde{\boldsymbol{u}}^T K_u \tilde{\boldsymbol{u}} - (1-\theta_e)(\boldsymbol{e}-\boldsymbol{\delta})^T K_k \tanh(\boldsymbol{e}-\boldsymbol{\delta}) \leq 0$$

Hence, applying Theorem 2.11 the system is ISS with respect to the inputs u_d and a. However, condition (4.42) can be satisfied only when the argument of the inverse hyperbolic tangent is less than 1, that is, when $(e - \delta)$ and v_r are small enough.

Boundedness of v_r , u_d , and a

Define a Lyapunov function for the sway velocity v_r

$$V_v = \frac{1}{2}v_r^2$$

In (3.6b), $u_r r$ can be viewed as a single, continuous input:

$$\dot{v}_r = f(t, v_r, u_r r) = -\frac{d_{v_r}}{m_v} v_r - \frac{m_u}{m_v} u_r r$$

The time derivative of the Lyapunov function is

$$\begin{aligned} \dot{V}_v &= v_r \dot{v}_r = -\frac{d_{v_r}}{m_v} v_r^2 + \frac{m_u}{m_v} v_r u_r r \\ &= -(1 - \theta_v) \frac{d_{v_r}}{m_v} v_r^2 - \theta \frac{d_{v_r}}{m_v} v_r^2 + \frac{m_u}{m_v} v_r u_r r \end{aligned}$$

where $0 < \theta_v < 1$. Then,

$$\dot{V}_v \le -(1-\theta)\frac{d_{v_r}}{m_v}v_r^2, \quad \forall \ |v_r| \ge \frac{m_u}{d_{v_r}\theta_v}|u_rr|$$

Hence, the conditions of Theorem 2.11 are satisfied, $||v_r(t)||_a \leq \gamma (||u_r(t)r(t)||_a)$ and in particular

$$\|v_r(t)\|_a \le \frac{m_u}{d_{v_r}\theta_v} \|u_r r\|_a$$
(4.43)

Consider now the dynamics of $\boldsymbol{u} = \boldsymbol{u}_d + \tilde{\boldsymbol{u}}$. From (4.33a)

$$\dot{\boldsymbol{u}} = \dot{\boldsymbol{u}}_d + \dot{\tilde{\boldsymbol{u}}} = -M^{-1} [K_d + D] \, \tilde{\boldsymbol{u}} - M^{-1} C(v_r) \boldsymbol{u} = -M^{-1} [K_d + D - C(v_r)] \, \boldsymbol{u} + M^{-1} [K_d + D] \, \boldsymbol{u}_d$$
(4.44)

Define a Lyapunov function

$$V_u = \frac{1}{2} \boldsymbol{u}^T \boldsymbol{u}$$

whose time derivative is

$$\dot{V}_{u} = \boldsymbol{u}^{T} \dot{\boldsymbol{u}} = -\boldsymbol{u}^{T} M^{-1} \left[K_{d} + D - C(v_{r}) \right] \boldsymbol{u} + \boldsymbol{u}^{T} M^{-1} \left[K_{d} + D \right] \boldsymbol{u}_{d}$$
(4.45)

If the matrix K_u in the first part of the proof is positive definite, this will be true also for the matrix $M^{-1}[K_d + D - C(v_r)]$. Then, defining $0 < \theta_u < 1$, when

$$\|u\| \ge \frac{1}{\theta_u} \| [K_d + D - C(v_r)]^{-1} M \| \| M^{-1} [K_d + D] \| \| \boldsymbol{u}_d \|$$

is verified, then

$$\dot{V}_u \leq -(1-\theta_u)\boldsymbol{u}^T M^{-1} \left[K_d + D - C(v_r)\right] \boldsymbol{u} \leq 0$$

and from Theorem 2.11

$$\|\boldsymbol{u}(t)\|_{a} \leq \frac{1}{\theta_{u}} \| [K_{d} + D - C(v_{r})]^{-1} M \| \| M^{-1} [K_{d} + D] \| \| \boldsymbol{u}_{d} \|_{a}$$
(4.46)

If \boldsymbol{u} is asymptotically bounded, the same must be true about its components u_r and r.

Then, it proceeds from (4.43) that

$$\|v_r(t)\|_a \leq \frac{m_u}{d_{v_r}\theta_v} \|u_r\|_a \|r\|_a \leq \frac{m_u}{d_{v_r}\theta_u\theta_v} \|\boldsymbol{u}(t)\|_a^2$$

with the bound $\|\boldsymbol{u}(t)\|_a$ depending on \boldsymbol{u}_d (see 4.46). From (4.32b),

$$\|\boldsymbol{u}_d\| \le \|\Delta^{-1}\| \left(\|K_k\| + \|\boldsymbol{v}_{c[t_0,\infty)}\| + \|\dot{\boldsymbol{p}}_{d[t_0,\infty)}\| \right)$$

As for the vector \boldsymbol{a} , considering that ||S(r)|| = |r|, and that there exist finite constants ϵ_1 and ϵ_2 such that

$$4\|B\tanh(\boldsymbol{e}-\boldsymbol{\delta})\| < \epsilon_1$$
$$4\|B(\boldsymbol{e}-\boldsymbol{\delta})\| < \epsilon_2,$$

for any $e \in \mathbb{R}^2$, from (4.36)

$$\|\boldsymbol{a}\| \le \|\Delta^{-1}\| \left[\|K_k\| \left(\epsilon_1 |r| + \epsilon_2 \|K_k\| + |v_r|\right) + \|\dot{\boldsymbol{v}}_{c_{xy}}\| + |r| \|\boldsymbol{v}_{c_{xy}}\| + |r| \|\dot{\boldsymbol{p}}_d(t)\| + \|\ddot{\boldsymbol{p}}_d(t)\| \right]$$

that is, $\|\boldsymbol{u}_d\|$ and $\|\boldsymbol{a}\|$ are bounded by a sum of bounded terms and are therefore bounded. Then, in accordance with Definition 2.9, the system is ISS with restrictions on the initial states $\boldsymbol{e}(0)$ and $v_r(0)$. To conclude the proof we now have to show that lifting the assumption that $\hat{\boldsymbol{v}}_c = \boldsymbol{v}_c$, that is, considering the dynamics of the observer, the closed-loop system is still ISS. This follows immediately from the fact that the closed-loop system can be viewed as the cascade of a globally asymptotically stable system (the observer) with output error $\tilde{\boldsymbol{v}}_c$, and an ISS system with input $\hat{\boldsymbol{v}}_c = v_c + \tilde{\boldsymbol{v}}_c$.

It is reasonable, following considerations of physical nature, to assume that the water current is bounded, while the bounds on $\dot{\mathbf{p}}_d$ and $\ddot{\mathbf{p}}_d$ are imposed when a certain mission is designed for the AUV. If $v_r(0)$ is small enough (or K_d is sufficiently large) for the matrix K_u to be positive definite then, as stated in the second part of the proof, $v_r(t)$ converges to a neighborhood of the origin, the radius of which is proportional to $||\mathbf{u}_d||^2$. The gain K_d of the inner-loop controller must then be high enough to relax the restriction on the initial condition of v_r , otherwise v_r might be pushed outside the stable zone. Then, given the bounds on \mathbf{v}_c and $\dot{\mathbf{p}}_d$ and the restriction on \mathbf{e} , the parameters μ and K_k must be chosen so to minimize the argument of the hyperbolic tangent in (4.42). This might be done by adopting a high value for μ , so K_d should be adjusted accordingly. Selecting a high gain K_k in the outer loop is helpful in rejecting the influence of \mathbf{v}_c and $\dot{\mathbf{p}}_d$, but increases \mathbf{u}_d , so a trade-off must be made in selecting its value. Notice that less conservative restrictions would be obtained analyzing the single scalar equations instead of the vectorial expressions.

4.6.2 Path-following controllers

The results stated in Theorem 4.1 apply to the control laws derived in Section 4.4.2, since in this case $\dot{\boldsymbol{p}}_d = \frac{\partial \boldsymbol{p}_d(\gamma)}{\partial \gamma} v_d$ and it is still reasonable to assume that the term is bounded. Therefore, Theorem 4.1 can be restated as follows.

Theorem 4.2. Consider the system described by (3.4) and (3.6) in closed-loop with the ocean current observer (4.6) and the control laws

$$\boldsymbol{\tau} = -K_d(\boldsymbol{u} - \boldsymbol{u}_d) + D\boldsymbol{u}_d \tag{4.47a}$$

$$\boldsymbol{u}_{d} = \Delta^{-1} \left(-K_{k} \tanh(\boldsymbol{e} - \boldsymbol{\delta}) - \hat{\boldsymbol{v}}_{c} + R^{T}(\boldsymbol{\psi}) \frac{\partial \boldsymbol{p}_{d}(\boldsymbol{\gamma})}{\partial \boldsymbol{\gamma}} \boldsymbol{v}_{d} \right)$$
(4.47b)

where

$$K_d = \begin{bmatrix} k_u & 0 \\ 0 & k_r \end{bmatrix}, \qquad K_k = \begin{bmatrix} k_x & 0 \\ 0 & k_y \end{bmatrix}$$

are positive definite, and let $\frac{\partial \mathbf{p}_d(\gamma)}{\partial \gamma} v_d$, $\frac{\partial^2 \mathbf{p}_d(\gamma)}{\partial \gamma^2} v_d$ and \mathbf{v}_c be bounded inputs. If the gains K_d and K_k are sufficiently large, then the system is ISS with restrictions on the initial states $\mathbf{e}(0)$ and $v_r(0)$

As for the path-following strategy II described in Section 4.5, the dynamics of the error z have to be taken into account, and a term $z\dot{z}$ has to be added to the derivative of the Lyapunov function (4.38). This however results in adding a negative term $-z^2$, so the properties of convergence of the closed-loop system are not changed. When the restriction on the initial condition e(0) is satisfied, the control variable $\ddot{\gamma}$ defined in (4.25b) is bounded, as it is the sum of bounded terms.

Theorem 4.3. Consider the system described by (3.4) and (3.6) in closed-loop with the ocean current observer (4.6) and the control laws

$$\boldsymbol{\tau} = -K_d(\boldsymbol{u} - \boldsymbol{u}_d) + D\boldsymbol{u}_d \tag{4.48a}$$

$$\boldsymbol{u}_{d} = \Delta^{-1} \left(-K_{k} \tanh(\boldsymbol{e} - \boldsymbol{\delta}) - \hat{\boldsymbol{v}}_{c} + R^{T}(\boldsymbol{\psi}) \frac{\partial \boldsymbol{p}_{d}(\boldsymbol{\gamma})}{\partial \boldsymbol{\gamma}} \boldsymbol{v}_{d} \right)$$
(4.48b)

$$\ddot{\gamma} = -k_z z + \frac{\partial v_d(\gamma)}{\partial \gamma} \dot{\gamma} + (\boldsymbol{e} - \boldsymbol{\delta})^T R^T(\psi) \frac{\partial \boldsymbol{p}_d(\gamma)}{\partial \gamma}$$
(4.48c)

where

$$K_d = \begin{bmatrix} k_u & 0\\ 0 & k_r \end{bmatrix}, \qquad K_k = \begin{bmatrix} k_x & 0\\ 0 & k_y \end{bmatrix}$$

are positive definite, and let $\frac{\partial^2 \mathbf{p}_d(\gamma)}{\partial \gamma^2} v_d$ and \mathbf{v}_c be bounded inputs. If the gains K_d and K_k are sufficiently large, then the system is ISS with restrictions on the initial states $\mathbf{e}(0)$ and $v_r(0)$.

4.7 Summary

In this chapter, trajectory-tracking and path-following controllers for an underactuated AUV were designed. To increase portability the motion control problems were divided into a dynamic task, assigned to an inner-loop controller, and a kinematic one, assigned to an outer-loop controller. Stability and convergence have been proven to hold, under some assumptions, in closed-loop. Notice that the results presented in this chapter are not based on the particular value of any of the physical parameters of the AUV, but on the structure of the general model described in Chapter 3. The control laws proposed are thus valid for a wide class of underwater vehicles.

Coordination

In this chapter we devise a decentralized control strategy to achieve coordination between multiple AUVs. After defining a convenient coordination error, based on the Laplacian of the graph associated to the communications network (Section 5.2), a coordination law is derived in Section 5.3 assuming that the vehicles communicate continuously. This assumption is lifted in Section 5.4 by introducing a logic based communications system, which in Section 5.5 is refined to take into account time-delays.

5.1 Introduction

In Chapter 4 control laws were derived to drive a single AUV along a desired path with a given speed profile. To do this the path was parametrized and a speed profile $v_d(\gamma)$ was assigned to the parameter's derivative $\dot{\gamma}$. Consider now a group of n AUVs (a *flock*): to achieve coordination between the elements of the group, a common speed profile v_L has to be assigned, so that the vehicles move along the given paths while holding a desired intervehicle formation pattern. The parameter γ of each vehicle can be seen as a coordination state such that coordination exists between two vehicles *i* and *j* if $\gamma_i(t) = \gamma_j(t)$. The problem is therefore decoupled in the motion control problem solved in Chapter 4, and a dynamic assignment task, that is the subject of this chapter, whose aim is to drive the coordination error between any two vehicles to zero.

5.2 Problem statement

Underwater communications and positioning rely heavily on acoustic systems, which are plagued with intermittent failures, latency, and multipath effects (Ghabcheloo *et al.*, 2006*a*). The controller should then be designed so to minimize the information flow. Because of the very nature of the intervehicle communications network a centralized control law, based on the knowledge of all the coordination parameters γ_i of the vehicles in the group, is not a practical solution to the problem, since every vehicle would have to receive, either directly or through other vehicles, information about the rest of the flock. The approach pursued in this thesis is instead a decentralized one, that takes into consideration the existing communication constraints: the correction speed \tilde{v}_{d_i} is determined only on the base of the measurements available to vehicle *i*, that is, the coordination states of the vehicles that communicate with *i*. To describe the communication topology it is a natural choice to resort to the graph theory notions introduced in Chapter 2. The vehicles in a flock are the vertices of a graph, the existing communication links are the edges, directed if the communication is unidirectional, undirected if the communication is bidirectional.

Consider a group of vehicles $\mathcal{I} := \{1, \ldots, n\}$. Let $\boldsymbol{\gamma} = [\gamma_1, \ldots, \gamma_n]^T$ be the vector containing the coordination states of the *n* vehicles, and \mathcal{N}_i denote the set of vehicles that vehicle *i* exchanges information with or, in the case of unidirectional communication, receives information from. The coordination problem can be formulated as follows (Aguiar *et al.*, 2007*b*)

Problem 5.1. For each vehicle $i \in \mathcal{I}$ derive a control law for the speed command \tilde{v}_{d_i} as a function of γ_i and γ_j , with $j \in \mathcal{N}_i$, such that for all $i, j \in \mathcal{I}$ $\gamma_i - \gamma_j$ approaches zero as $t \to \infty$, and the formation travels at an assigned speed v_L , that is, $|\dot{\gamma} - v_L|$ tends to zero.

Define an error vector

$$\boldsymbol{\xi} = L_D \boldsymbol{\gamma} \tag{5.1}$$

where L_D is the normalized Laplacian of expression (2.24), obtained associating a *commu*nication graph (Ghabcheloo *et al.*, 2006*b*) to the AUV formation. The assumption is made that the communication topology does not change in time, *i.e.*, the Laplacian is constant. The *i*-th element of the vector is

$$\xi_i = \gamma_i - \frac{1}{|\mathcal{N}_i|} \sum_{j \in \mathcal{N}_i} \gamma_j$$

that is, the sum of the coordination errors between vehicle *i* and the vehicles that communicate with it. The single variable ξ_i embodies the communication constraints of the network and can be used for control purposes (Ghabcheloo, 2007). This fact plays a key role in the computation of a decentralized coordination control law.

Theorem 2.15 states that 1 belongs to the kernel of L_D , so when all the vehicles are in coordination, γ is a multiple of 1 and consequently the error vector $\boldsymbol{\xi}$ is null. If any vehicle is isolated from the rest of the formation the Laplacian has m null eigenvalues (Theorem 2.16), corresponding to the rigid motions of the m isolated elements of the flock. In the following, only strongly connected formations will be considered. The kernel has therefore dimension 1 and is spanned by the unit vector 1. This is a crucial result, as it guarantees that the error $\boldsymbol{\xi}$ is null *only* when the vehicles are in coordination. Furthermore, we will assume that the communication is bidirectional. This implies that the Laplacian is symmetric and that the left eigenvector associated to the null eigenvalue is $\mathbf{1}^T$.

5.3 Coordination with continuous communication

The dynamics of the coordination error defined in (5.1) are described by

$$\dot{\boldsymbol{\xi}} = L_D \dot{\boldsymbol{\gamma}}$$

Consider the path-following controller of Section 4.4.2. The speed profile was assigned to each vehicle identically through the equality $\dot{\gamma} = v_d(\gamma)$, which substituted into the foregoing expression yields

$$\boldsymbol{\xi} = L_D \boldsymbol{v}_d \tag{5.2}$$

where v_d is the vector whose elements are the desired speeds of the vehicles in the flock. The key idea in designing the coordination controller is to introduce a control variable in the form of a correction term \tilde{v}_d that is added to the desired speed:

$$\boldsymbol{v}_d = \boldsymbol{v}_L + \tilde{\boldsymbol{v}}_d \tag{5.3}$$

where \boldsymbol{v}_L is the speed profile assigned to the path.

Proposition 5.1. Consider a formation of n vehicles, each guided by the motion control laws (4.47) along a path parametrized by γ_i , and let L_D be the normalized Laplacian of a graph that describes the inter-vehicle communications network. Assume that any two neighboring vehicles communicate continuously. Then, the decentralized control law

$$\tilde{\boldsymbol{v}}_d = -k_{\boldsymbol{\xi}} \tanh\left(L_D \boldsymbol{\gamma}\right) \tag{5.4}$$

where k_{ξ} is a sufficiently large positive constant, makes the coordination error ξ and the speed profile v_d converge to neighborhoods respectively of the origin and of v_L . Morover, if $v_L = k\mathbf{1}$ then ξ and v_d converge asymptotically to $\mathbf{0}$ and to v_L , respectively, and Problem 5.1 is solved.

Proof. Substituting the control law (5.4) in (5.3), and the resulting expression in (5.2), yields

$$\dot{\boldsymbol{\xi}} = L_D \boldsymbol{v}_L - k_{\boldsymbol{\xi}} L_D \tanh\left(\boldsymbol{\xi}\right) \tag{5.5}$$

Let v_L be considered as a perturbation input. Then $\boldsymbol{\xi} = \mathbf{0}$ is an equilibrium point of the unforced closed-loop system. Define a Lyapunov function

$$V_{\boldsymbol{\xi}} = \frac{1}{2} \boldsymbol{\xi}^T \boldsymbol{\xi}$$

whose time derivative is

 $\dot{V}_{\boldsymbol{\xi}} = \boldsymbol{\xi}^T \dot{\boldsymbol{\xi}}$

From (5.5)

$$\dot{V}_{\boldsymbol{\xi}} = \boldsymbol{\xi}^T L_D \boldsymbol{v}_L - k_{\boldsymbol{\xi}} \boldsymbol{\xi}^T L_D \tanh \boldsymbol{\xi}$$

= $-(1 - \theta + \theta) k_{\boldsymbol{\xi}} \boldsymbol{\xi}^T L_D \tanh \boldsymbol{\xi} + \boldsymbol{\xi}^T L_D \boldsymbol{v}_L$ (5.6)

with $0 \le \theta \le 1$. In Chapter 4 it had been assumed that v_L is bounded. Therefore, provided that the gain k_{ξ} is large enough, there always exists some ξ for which

$$\|\boldsymbol{\xi}\| \ge \left\| \operatorname{arctanh}\left(\frac{1}{\theta k_{\xi}} \boldsymbol{v}_{L}\right) \right\|$$
(5.7)

As the Laplacian L_D is semipositive definite, from (5.6) and (5.7) we obtain

$$\dot{V}_{\boldsymbol{\xi}} \leq -(1-\theta)k_{\boldsymbol{\xi}}\boldsymbol{\xi}^T L_D \tanh \boldsymbol{\xi} \leq 0$$

The foregoing inequality is trivially satisfied not only at the origin but also, from the properties of the Laplacian, where $\boldsymbol{\xi}^T = k \mathbf{1}^T$. However, this can never be verified, because it would imply $\boldsymbol{\gamma}^T L_D = \mathbf{1}^T$ and $\mathbf{1}^T$, that spans the kernel of L_D , cannot belong also to its image. Then $\dot{V}_{\boldsymbol{\xi}}$ is negative definite and, applying Theorem 2.11, $\boldsymbol{\xi}$ is ISS with respect to \boldsymbol{v}_L . This means that

$$\|oldsymbol{\xi}\|_a \leq ext{arctanh}\left(rac{1}{ heta k_{\xi}}\|oldsymbol{v}_L\|_a
ight)$$

and

$$\| ilde{oldsymbol{v}}_d\|_a \leq rac{1}{ heta} \|oldsymbol{v}_L\|_a$$

so from (5.3)

$$\|oldsymbol{v}_d - oldsymbol{v}_L\|_a \leq \|oldsymbol{ ilde v}_d\|_a \leq rac{1}{ heta}\|oldsymbol{v}_L\|_a$$

that is, v_d is bounded to a neighborhood of v_L . Notice that selecting a higher gain k_{ξ} reduces the size of the neighborhood to which ξ converges, but not the bound on v_d . However, if $v_L = k\mathbf{1}$, *i.e.*, if the same speed profile is assigned to every vehicle, then v_L disappears from (5.6) and the equilibrium points $\boldsymbol{\xi} = \mathbf{0}$ and $v_d = v_L$ are globally asymptotically stable. \Box

The control law (5.4) satisfies the requirement formulated in Chapter 4 that the speed profile $\mathbf{v}_d = \mathbf{v}_L + \tilde{\mathbf{v}}_d$ be bounded. The results of Proposition 5.1 apply also to the second path-following strategy of Section 4.5, the only difference being that $\dot{\gamma} = \mathbf{v}_d$ is not satisfied identically. Instead $\dot{\gamma}$ converges to a neighborhood of \mathbf{v}_d , the overall system being ISS with restrictions on the initial states $\mathbf{e}(0)$ and $v_r(0)$. We can thus conclude that the overall closed-loop system composed by n AUVs, each one equipped with one of the path-following controllers of Section 4.6.2, and the coordination law (5.4), is ISS (with restrictions) and solves the coordination problem 5.1.



Figure 5.1: Coordination control with continuous communication

5.4 Coordination with discrete communication

The coordination controller designed in Section 5.3 relies on the continuous exchange of information between the vehicles in the formation. All vehicle continuously broadcast their parametrization state, and the parameter vector is available to every vehicle at any instant in time (see Fig. 5.1). Underwater communications however are characterized by low bandwidths that only allow the exchange of data to take place at discrete instants of time. In (Aguiar and Pascoal, 2007*a*) a logic-based communications strategy is proposed, that takes into account both the fact that communications do not occur in a continuous manner and the cost of exchanging information. In between communications, that are regulated by a supervisory logic, each vehicle runs estimations of the coordination states of the rest of the flock (see Fig. 5.2). This is done through synchronized estimation blocks, identical for every vehicle, that have the following expression, based on (5.3) and (5.4):

$$\dot{\hat{\boldsymbol{\gamma}}} = \boldsymbol{v}_L(\hat{\boldsymbol{\gamma}}) - k_{\boldsymbol{\xi}} \tanh\left(L_D \hat{\boldsymbol{\gamma}}\right)$$

Every agent runs, amongst the others, an estimate of its own state. It is by comparing the actual value of its state with this estimate that a vehicle decides when to communicate with the vehicles in its neighborhood. If, at a certain instant t_k , $|\gamma_i - \hat{\gamma}_i| \ge \epsilon^2$, then vehicle *i* broadcasts the value of γ_i . Assuming that no delays affect the communication links, each



Figure 5.2: Coordination control with logic-based communication

vehicle updates its estimate instantly, so that

$$\hat{\boldsymbol{\gamma}}_i(t_k) = \boldsymbol{\gamma}_i(t_k)$$

Remembering the expression of the normalized Laplacian (2.24), the control law (5.4) becomes then

$$\tilde{\boldsymbol{v}}_d = -k_{\boldsymbol{\xi}} \tanh\left(\boldsymbol{\gamma} - D^{-1}A\hat{\boldsymbol{\gamma}}\right) \tag{5.8}$$

where it has been explicited that the correction term for every vehicle is the sum of a term that depends on the coordination state of the vehicle itself, which is available in every instant, and a term built on the estimations of the states of the other vehicles. In the instants between communication, defining the estimation error $\tilde{\gamma} = \gamma - \hat{\gamma}$, inequality (5.6) becomes

$$\dot{V}_{\boldsymbol{\xi}} = -(1-\theta+\theta)k_{\boldsymbol{\xi}}\boldsymbol{\xi}^{T}L_{D} \tanh\left(\boldsymbol{\xi}+D^{-1}A\tilde{\boldsymbol{\gamma}}\right) + \boldsymbol{\xi}^{T}L_{D}\boldsymbol{v}_{L}$$

As $|\tilde{\gamma}|$ is bounded by ϵ^2 it can be dominated by ξ , so the system is ISS with respect to $\tilde{\gamma}$. Selecting a lower tolerance ϵ^2 reduces the neighborhood of the origin to which ξ converges but increases the number of messages exchanged between vehicles. Notice that although the control signal for each vehicle is based on the states of its neighbors, every agent runs an estimation of the states of the whole flock. In the absence of delay, the updates of the



Figure 5.3: Communication logic 1: At time t_k vehicle 1 sends a message to vehicle 2 (a). Vehicle 2 receives the message at $t_k + \tau$ and immediately replies (b). Then, at $t_k + 2\tau$, it updates its estimate (c). Vehicle 1 only updates the estimate at $t_k + 2\tau + \tilde{\tau}$ (d).

coordination states can be retransmitted across the formation and reach instantly every vehicle (the graph is connected), so every vehicle has access to the same estimates, as would happen if one single centralized estimator was shared by all the vehicles.

5.5 Coordination with time-delays

Underwater communications rely on acoustic systems that are characterized by low bandwidth and short range. The communication channels are plagued with intermittent failures, latency and multi-path effects, and delay is introduced both by the processing required and by the distance between vehicles. Assume that at time t vehicle i broadcasts its coordination state. Vehicles j and k will receive the message at $t + t_j$ and $t + t_k$ respectively. If the three vehicles were to update their estimate of γ_i as soon as they receive (or send, in the case of i) the message, then the estimator blocks would cease to be synchronized. The communication strategy of Section 5.4 must then be modified, taking into account the network topology, and the update must take place so to keep the estimators always synchronized.
A solution to the estimation problem in the presence of time-delays, proposed in (Aguiar and Pascoal, 2007*a*), requires for each vehicle to be equipped with as many independent estimation blocks as the number of its neighbors. For every communication link there are therefore two estimators that have to be kept synchronized. A single vehicle runs $|\mathcal{N}_i|$ different estimates of γ . Let γ_i^{jk} denote the estimate of γ_i run by vehicle j on the estimator associated with the link between j and k. If, at a certain instant t_k , $|\gamma_i - \hat{\gamma}_i^{ij}| \ge \epsilon^2$, then vehicle i sends a message containing the actual value γ_i and the time t_k to vehicle j. Vehicle j receives the message at $t_k + \tau$ but does not update its estimate of γ_i instantly. Instead, it sends a "received" message back to i, and only executes the update in synchronization with i.

In (Aguiar and Pascoal, 2007*a*) *j* updates its estimate at $t_k + 2\tau$, while *i* does the same upon reception of the reply. After the update, the estimation error for γ_i on the link *ij* is

$$\boldsymbol{\gamma}_i - \hat{\boldsymbol{\gamma}}_i^{ij} = \int_{t_k}^{t_k + 2\tau} \dot{\boldsymbol{\gamma}}_i(t) dt$$

which is bounded assuming that the time-delay is bounded. This strategy is based on the assumption that the delay on a communication channel is the same in both directions. A small difference $\tilde{\tau}$ however will always be present, so an error exists also between the estimates of the two vehicles over the same link, that is

$$\hat{\boldsymbol{\gamma}}_{i}^{ji} = \hat{\boldsymbol{\gamma}}_{i}^{ij} + \int_{t_{k}+2\tau}^{t_{k}+2\tau+\tilde{\tau}} \dot{\hat{\boldsymbol{\gamma}}}_{i}^{ji}(t) dt$$

An example is given in Fig. 5.3.

An alternative update logic is based on a statistical evaluation of the time that is required for the message to be sent and for the answer to be received. If at time t_k vehicle *i* needs to communicate γ_i to vehicle *j*, it estimates a maximum delay τ_{max} and sends a message containing t_k, γ_i and τ_{max} . If *i* receives an answer from *j* before $t_k + \tau_{max}$ then both vehicles update their estimates at the instant $t_k + \tau_{max}$ (see Fig. 5.4). If the answer is not received within the limit time, instead, the message is considered lost and a new message is sent, with the up-to-date value of γ_i . The limit τ_{max} has to be choosen so to drive the probability of a lost message under a defined error margin. As the time between the sending of a message and the update is maximized, when compared with the previous strategy this one introduces a greater error between the actual state and the estimate:

$$\boldsymbol{\gamma}_i - \hat{\boldsymbol{\gamma}}_i^{ij} = \int_{t_k}^{t_k + \tau_{max}} \dot{\boldsymbol{\gamma}}_i(t) dt$$



Figure 5.4: Communication logic 2: At time t_k vehicle 1 sends a message to vehicle 2 (a). Vehicle 2 receives the message at $t_k + \tau$ and immediately replies (b). Both vehicles update their estimate at the same instant $t_k + \tau_{max}$ (c).

However, it assures that the estimates over the same link are synchronized, that is

$$oldsymbol{\gamma}_i - \hat{oldsymbol{\gamma}}_i^{ij} = oldsymbol{\gamma}_i - \hat{oldsymbol{\gamma}}_i^{ji}$$

In the occurrance that j receives the message but i does not receive the reply, only j makes an update. The difference introduced between the two estimates is however corrected as isends immediately a new message.

The control law (5.8) is still valid in the presence of time-delays. If a vehicle has more than one estimation block, however, $\hat{\gamma}$ will be a combination of all the estimates run by that vehicle. In the two examples of Fig. 5.3 and Fig. 5.4, vehicle 1 has two estimator blocks. Between the two available, the most accurate estimate of γ_2 is the one of the estimator synchronized with vehicle 2, that will therefore be chosen as the value to use in the control law. For the same reason, the estimate of the block synchronized with vehicle 3 will be adopted as γ_3 , so that

$$\tilde{\boldsymbol{v}}_{1} = -k_{\xi} \tanh\left(\boldsymbol{\gamma}_{1} - \frac{1}{2}\hat{\boldsymbol{\gamma}}_{2}^{12} - \frac{1}{2}\hat{\boldsymbol{\gamma}}_{3}^{13}\right)$$
(5.9a)

$$\tilde{\boldsymbol{v}}_2 = -k_{\xi} \tanh\left(\boldsymbol{\gamma}_2 - \hat{\boldsymbol{\gamma}}_1^{21}\right) \tag{5.9b}$$

$$\tilde{\boldsymbol{v}}_3 = -k_{\xi} \tanh\left(\boldsymbol{\gamma}_3 - \hat{\boldsymbol{\gamma}}_1^{31}\right) \tag{5.9c}$$

For the same reason, when the estimates over links 12 (or 13) are updated, the value of $\hat{\gamma}_3$ (of $\hat{\gamma}_2$) used is $\hat{\gamma}_3^{13}$ (or $\hat{\gamma}_3$) that vehicle 1 knows and communicates to vehicle 2 (or 3) along with γ_1 and t_k . If the communication graph is a tree, the strategy used to obtain (5.9) can be generalized. Consider a vehicle *i*: the path connecting it to any vehicle *j* that does not belong to its neighborhood passes through only one of its neighbors, *k*. Then, the value chosen by *i* for the control law and for estimation updates will be $\hat{\gamma}_j^{ik}$, generated by the estimator synchronized with *k*. Future research will consider other strategies to weigh and combine the estimates available to one vehicle.

5.6 Summary

In this chapter Lyapunov control techniques and graph theory have been brought together to design a decentralized controller that drives multiple autonomous vehicles along a path while maintaining a desired inter-vehicle spatial pattern. By associating a logic-based communications system to the coordination control we addressed the fact that communications between agents only occur at discrete instants in time, and proposed a solution to tackle the problem of time-delay in communications, that is especially challenging in underwater applications of formation control.

Simulation Results

This chapter illustrates via computer simulations the performance of the control strategies devised in this thesis. The first simulation (Section 6.1) compares the trajectory-tracking and the path-following controllers when driving the vehicle along the same path. In the second simulation (Section 6.2) the robustness of the path-following controller is tested by introducing sensor noise and water current. In Section 6.3 we simulate a coordinated path-following mission. Section 6.4 analises a simulation of the same mission in the presence of water current and variable time-delays. All the parameters and the settings for the simulations in this chapter are reported in Table 6.1.

6.1 Trajectory-tracking and path-following

In this first simulation (see Fig. 6.1) an AUV was required to converge to, and then to move along, a straight line, from (30,30) to (100,100).

In the case of trajectory-tracking, the virtual target progresses along the desired path with fixed velocity v_d , starting from (30,30). The AUV rotates, without moving forward, to point the virtual target, until this reaches the point in the path closest to the vehicle. Until that moment, the term in the control law (4.32b) relative to the position error and the one relative to the time derivative of the position of the virtual target are opposed in sign, and balance each other. Once the virtual target reaches the closest point, the vehicle surges forward, reaches the path and then aligns itself to it to move along it. Note that this "good behavior" is due to boundedness of the hyperbolic tangent function in (4.32b). Without this function, the transient exhibited by the AUV would have been considerably less smooth.

In the case of path-following (4.47b) the starting value of the parameter γ is the one corresponding to the point of the path closest to the vehicle, so the initial position error is smaller. With the first path-following strategy the virtual target progresses along the path with fixed velocity v_d , and the AUV chases it, gradually aligning its velocity with the desired path. The trajectory followed by the AUV is therefore smoother. However, convergence to the path is achieved more slowly.

When adopting the second path-following strategy (4.48c) since the evolution of the



Figure 6.1: Motion of an AUV and the relative position error in the case of trajectory-tracking (a), path-following (strategy I) (b) and path-following (strategy II) (c).

position of the virtual target depends also on the position error, p_d remains fixed at the point of the path closest to the AUV until the AUV reaches the path. The trajectory of the AUV is similar to the one obtained with trajectory-tracking, as in both cases the AUV approaches the path almost perpendicularly, and only after having reached it it aligns its velocity to the direction to follow.

Notice that if the point in the path closest to the AUV's starting position is the beginning of the path, trajectory-tracking controller and the path-following strategy I yield the same results, as the virtual target's initial position and its rate of progression are the same in both cases.

6.2 Robustness of the path-following controller

The stability and convergence conditions resulting from the analysis, in Section 4.6, of the closed-loop control system in absence of the sway velocity sensor, are very conservative. Two reasons for this are the following: (i) we have considered the vectorial dynamic equations, instead of the single scalar equations, and (ii) we did not take into account the fact that the damping forces increase with velocity. To assess the robustness of the path-following controllers, a mission was simulated in which an AUV is required to follow a path, made of a straight line followed by a circle, in the presence of sensor noise, affecting the measurements of both position and velocity, strong water current and saturation of the thrusters. Moreover, at the initial time the AUV faces away from the path. The results obtained were very similar for both path-following strategies; we report the ones relative to strategy II.

As shown in Fig. 6.2, the controller exhibits very good robustness properties. The AUV's trajectory in the simulation with disturbances (b) is very close to the one of the case without disturbances (a), and the position error (c) is bounded to a small value. The control signals of the inner loop (u_d (d) and r_d (e)) and the AUV velocities (d,e, and f) are also bounded. There is a peak in yaw speed at the beginning of the simulation, when the AUV has to rotate 180° to point the virtual target. The virtual target doesn't move, *i.e.*, the parameter γ is constant, both at the beginning, when the position error is large, and after the end of the path has been reached (Fig. 6.3). The observer designed in (4.3) produces a good estimate of the water current speed and direction, which in this simulation were made to vary. Notice that when it reaches the end of the path the AUV stops by facing the water current and balancing its action with its thrusters.



Figure 6.2: Path-following in the presence of sensor noise, water current and thruster saturation: trajectory of the AUV with no disturbances (a) and with disturbances (b), evolution of the position error (c) and AUV velocities (d,e, and f).



Figure 6.3: Path-following in the presence of sensor noise, water current and thruster saturation: evolution of the parameter γ and of its time derivative and estimated water current

6.3 Coordinated path-following

In this simulation three AUVs, with the communications network topology shown in Fig. 5.1 were required to follow a path while maintaining a spatial formation.

In the absence of time-delays, adopting the first path-following strategy the estimator blocks onboard the AUVs are identically synchronized. The vehicles communicate just once at the beginning of the path to synchronize their initial values of γ (as do vehicle 2 and vehicle 3 in Fig. 6.5). To force communication and test the robustness of the coordination algorithm, between 225 s and 300 s the time derivative of the parameter γ of vehicle 3 is made to assume a fixed value, thus introducing a difference with the estimated value and increasing the coordination error (Fig. 6.4). As shown in Fig. 6.5 when this happens vehicle 3 starts to communicate with the other two, sending the actual value of γ_3 every time that the difference with the estimated value passes a selected tolerance. Vehicles 1 and 2 adjust their estimations and the evolution of their respective virtual targets changes acordingly. The whole formation slows down until 300 s, and then finishes moving along the path at the desired speed v_L .

The first path-following strategy completely decouples the path-following problem for each vehicle and the coordination problem for the formation. In other words, it synchronizes the virtual targets, while the coordination of the vehicles depends on how well each vehicle tracks the corresponding target. To address the situation in which the formation has to be maintained when one of the vehicles has a difficulty in following the target, the solutions are either to adopt the second path-following strategy, or to design a supervisory logic that makes the vehicle that is affected by the problem modify the dynamic evolution of its coordination parameter, and therefore transmit its new coordination behavior to the other vehicles in the formation.

6.4 Coordinated path-following with communication delay

The simulation of Section 6.3 was repeated, adopting the second path-following strategy, in the presence of time-delays, sensor noise, water current and thruster saturation. Fig. 6.6 shows that coordination is achieved even in the presence of disturbances: both the position errors of each one of the vehicles and the coordination error are bounded to small values. The effect of time-delays on the synchronization of the estimators is illustrated in Fig. 6.7. At the initial instant, γ_2 and γ_3 are set to values that differ from zero more than the tolerance (in this case, 5). The two vehicles immediately communicate their states to vehicle 1. The reset of $\hat{\gamma}_2$ on the communication link 1-2, and of $\hat{\gamma}_3$ on link 1-3, occur more or less (the time-delay is variable) after 5 seconds. Vehicle 1 then communicates these two values, and the value of γ_1 , over the links opposite to the ones over which the values were received. The corresponding resets occur at 10 s for vehicle 2, at 11 s for vehicle 3 and at 12.5 for vehicle 1.



Figure 6.4: Coordinated path-following: trajectories and position errors of the AUVs and coordination error



Figure 6.5: Coordinated path-following: evolution of the parameters γ and communication signals



Figure 6.6: Coordinated path-following in the presence of time-delays: trajectories and position errors of the AUVs and coordination error



Figure 6.7: Coordinated path-following in the presence of time-delays: evolution of the parameters γ and communication signals

Simulation	6.1	6.2	6.3	6.4
Inner loop				
k_u	4000			
k_r	4000			
Outer loop				
δ	-0.5			
k_x	1			
k_y	0.6			
k_z	1			
Water current observer				
$K_{p_{obs}}$	Ι			
$K_{c_{obs}}$	Ι			
Coordination				
k_{ξ}	-	- 0.5		
Water current				
$\ oldsymbol{v}_c\ $	0	$0.5 \mathrm{m/s}$	0	0.5 m/s
ϕ_c	0	-45°	0	-45°
$\sigma^2(\ oldsymbol{v}_c\)$	0	0.01	0	0.01
$\sigma^2(\phi_c)$	0	0.01	0	0.01
Sensor noise				
$\sigma^2(oldsymbol{p})$	0	0.5		
$\sigma^2(oldsymbol{u})$	0	0.5		
Communication delay				
$\bar{ au}$	-	-	0	$5 \mathrm{s}$
$\sigma^2(\bar{\tau})$	-	-	0	0.5
Thruster saturation				
F _{max}	-	$1500~\mathrm{N}$	-	1500 N

Table 6.1: Parameters and settings of the simulations

6.5 Summary

The simulations illustrated in this chapter show that the control strategies proposed in this thesis exhibit a good performance in a wide range of scenarios. The results confirm that the stability properties assessed in Chapters 4 and 5 hold for a wide range of initial conditions. Furthermore, it is shown that the closed-loop systems are robust to sensor noise and water current and are not affected relevantly by the introduction of thruster saturation.

Conclusions and Further Research

7.1 Summary

Motivated by numerous mission scenarios, this thesis addressed the problem of motion control of multiple autonomous underwater vehicles. Borrowing from Lyapunov based techniques and graph theory we designed decentralized controllers that drive a flock of AUVs along a desired path, while maintaining a specified inter-vehicle formation. The problem was decoupled into the motion control task of making every vehicle follow a virtual target along its corresponding path, and a dynamic assignment task of adjusting the speed of the virtual targets so to achieve coordination.

In Chapter 4 we proposed different strategies for motion control, the common thread being the separation between an inner-loop, that makes the vehicle follow a speed reference, and an outer-loop that regulates this reference so that the vehicle tracks the virtual target. The resulting control laws are valid for a wide range of underactuated AUVs.

In Chapter 5 we derived a decentralized coordination law based on a framework that involves the concept of graph Laplacian. To address the fact that communication bandwidth is very limited in underwater applications, we devised a supervisory logic that minimizes the need for inter-vehicle communication.

The performance of the architecture resulting from bringing together the motion control and the coordination strategies was assessed through mathematical analysis and simulations. The system exhibited good behaviour in terms of stability, convergence, and robustness to disturbances.

7.2 Future directions

The problems addressed while developing this work cover a vast number of fields. Some of the results obtained are preliminary, and point out to possible avenues for future research.

Communication algorithms Whether all the vehicles *achieve consensus*, *i.e.*, their estimates converge to a common value, depends on the network topology and on the communication logic. *Consensus seeking* is the area of research that studies the techniques that allow

n autonomous agents to achieve consensus (Lin *et al.*, 2004; Olfati-Saber and Murray, 2004; Moreau, 2005). Many applications of coordinated control rely on the assumption that the convergence speed of consensus seeking is fast enough. For this reason, a widespread effort is being devoted to investigate methods that improve the convergence speed, such as finding the optimal weights associated with every communication link or using random rewiring to change the topology, but there are still many difficulties in practical implementation.

Supervisory logic This thesis did not explicitly address the problem of vehicle failures and avoidance of obstacles and inter-vehicle collision, situations in which one of the vehicles may be required to move with a different speed, or along a different path, than the ones assigned by the mission. Each vehicle should be afforded with the capability, at logic level, to alter its own parametrized path. The communication logic described in Chapter 5 would then assure that the other vehicles are aware of the change and that the coordination error is bounded.

Coordinated tracking The coordination strategies proposed in this thesis require the parametrization of the AUVs' path. To benefit from the advantages of cooperation between a network of agents also in trajectory-tracking missions, in which the path is not known *a priori*, such as following a target whose position is known through sensing, or facing the situations described in the previous point, a strategy has to be devised to parametrize the path online and assign to each vehicle parametrization states that can be synchronized to achieve coordination. While tracking a target, for example, a leader vehicle could sense its position at discrete intervals of time and add successive parametrized segments to its desired path. The information to be sent to the other vehicles could be reduced to the segment's length and direction, sufficient for the each of the other vehicles to parametrize its own segment in a way to keep the desired formation.

Cooperative navigation Cooperative navigation has been (partially) addressed for land robots or moving nodes in sensor networks. The main idea is to make use of the fact that each individual member of the group could benefit from navigation information obtained from other members. For underwater vehicles, cooperative navigation is considerably more challenging but very attractive. Only few vehicles are needed to maintain an accurate estimate of their positions through sophisticated (and expensive) navigation sensors (e.g., Doppler velocity logger and inertial navigation system). The other ones (a much larger group) can have less sophisticated navigation suites.

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